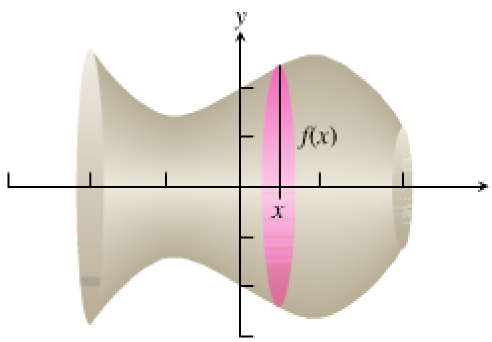
Circular Cross Sections

The only thing that changes when the cross sections of a solid are circular is the formula for $A(x)$. Many such solids are **solids of revolution,** as in the next example.

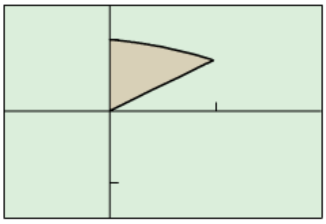
EXAMPLE 2 A Solid of Revolution

The region between the graph of $f(x) = 2 + x \cos x$ and the x-axis over the interval $[-2, 2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

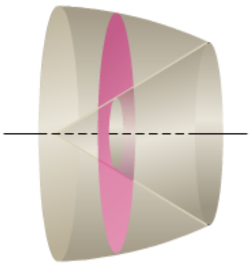


EXAMPLE 3 Washer Cross Sections

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.



$[-\pi/4, \pi/2]$ by $[-1.5, 1.5]$



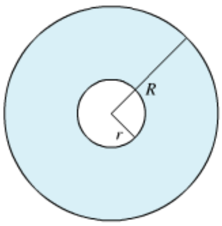
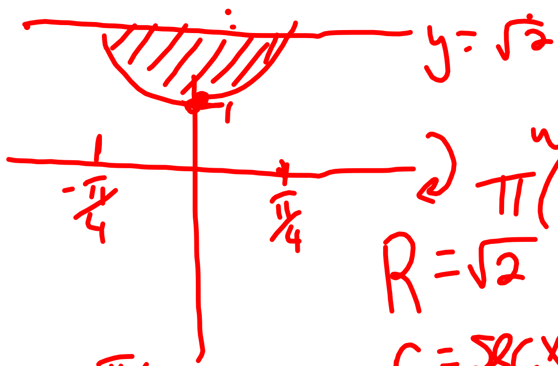


Figure 8.22 The area of a washer is $\pi R^2 - \pi r^2$. (Example 3)

19. $y = \sec x \quad y = \sqrt{2}$

$[-\pi/4, \pi/4]$



wasner

$\pi(R^2 - r^2)$

$R = \sqrt{2}$

$r = \sec x$

$$V = \pi\int_{-\pi/4}^{\pi/4} 2 - \sec^2 x\,dx$$

$$\pi\left[2x - \tan x \Big|_{-\pi/4}^{\pi/4}\right]$$

$$\pi\left[\left(\frac{\pi}{2} - \tan\frac{\pi}{4}\right) - \left(-\frac{\pi}{2} - \tan\frac{-\pi}{4}\right)\right.$$

$$\left(\frac{\pi}{2} - 1\right) - \left(\frac{-\pi}{2} - -1\right)$$

$$\pi\left[\frac{\pi}{2} - 1 + \frac{\pi}{2} - 1\right]$$

$$\pi\left[\pi - 2\right]$$

$$\pi^2 - 2\pi \quad u^3$$

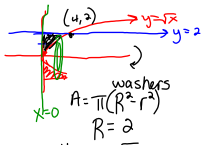
29. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about

(a) the x-axis. (b) the y-axis.

(c) the line $y = 2$. (d) the line $x = 4$.

a) x axis

$\sqrt{x} = 2$
$x = 4$



washers

$A = \pi(R^2 - r^2)$

$R = 2$

$r = \sqrt{x}$

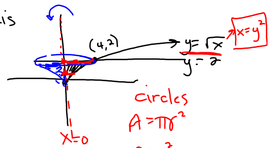
$V = \pi \int_0^4 4 - x \, dx$

$\pi \left[4x - \frac{x^2}{2} \Big|_0^4 \right]$

$\pi \left[(16 - 8) - (0) \right]$

$\boxed{8\pi u^3}$

b) y axis



circles

$A = \pi r^2$

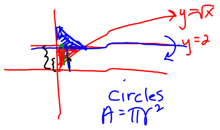
$r = y^2$

$V = \pi \int_0^2 (y^2)^2 \, dy$

$V = \pi \int_0^2 y^4 \, dy$

$\pi \left[\frac{y^5}{5} \Big|_0^2 \right] = \boxed{\frac{32}{5}\pi u^3}$

c) y=2



circles

$A = \pi r^2$

$r = 2 - \sqrt{x}$

$V = \pi \int_0^4 (2 - \sqrt{x})^2 \, dx$

$V = \pi \int_0^4 4 - 4\sqrt{x} + x \, dx$ ($-4x^{1/2}$)

$\pi \left[4x - 4x^{3/2} \cdot \frac{2}{3} + \frac{x^2}{2} \Big|_0^4 \right]$

$\pi \left[4x - \frac{8}{3}\sqrt{x^3} + \frac{x^2}{2} \Big|_0^4 \right]$

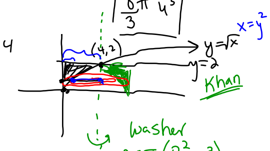
$\pi \left[\left(16 - \frac{64}{3} + 8\right) - (0) \right]$

$\pi \left[24 - \frac{64}{3} \right]$

$\pi \left[\frac{72}{3} - \frac{64}{3} \right]$

$\boxed{\frac{8\pi}{3} u^3}$

d) x=4



washer

$A = \pi(R^2 - r^2)$

$R = 4$

$r = 4 - y^2$

$V = \pi \int_0^2 4^2 - (4 - y^2)^2 \, dy$

$16 - (16 - 8y^2 + y^4)$

$\cancel{16} - \cancel{16} + 8y^2 - y^4$

$\pi \int_0^2 8y^2 - y^4 \, dy$

$\pi \left[\frac{8y^3}{3} - \frac{y^5}{5} \Big|_0^2 \right]$

$\pi \left[\left(\frac{5 \cdot 64}{5 \cdot 3} - \frac{32 \cdot 3}{5 \cdot 3} \right) - (0) \right]$

$\pi \left[\frac{320 - 96}{15} \right]$

$\boxed{\frac{224}{15} \pi u^3}$

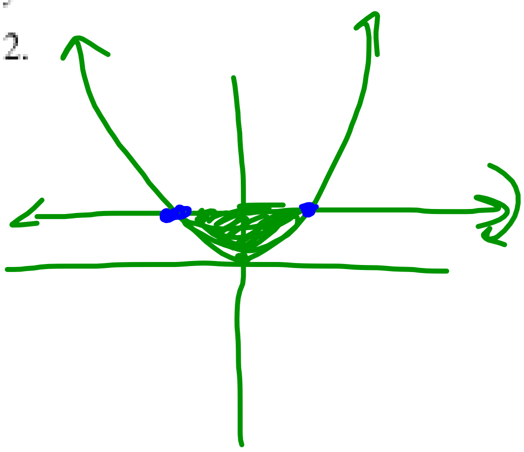
30. Find the volume of the solid generated by revolving the triangular region bounded by the lines $y = 2x$, $y = 0$, and $x = 1$ about

(a) the line $x = 1$. **(b)** the line $x = 2$.

31. Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about

(a) the line $y = 1$. **(b)** the line $y = 2$.

(c) the line $y = -1$.



Homework 8.3:
Day 1 1-7 odd,39 (cross sections)
Day 2 11,16,19,22(circles and washers)
Day 3 9,23,27,41(rotate around the y-axis)