Section 8.2

Areas in the Plane

What you'll learn about

- Area Between Curves
- Area Enclosed by Intersecting Curves
- Boundaries with Changing Functions
- Integrating with Respect to y
- Saving Time with Geometric Formulas

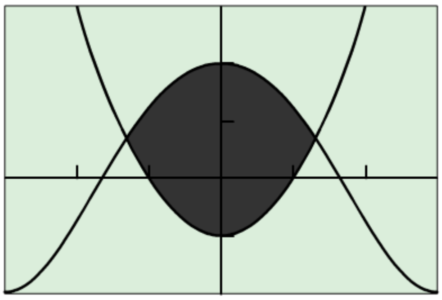
…and why

The techniques of this section allow us to compute areas of complex regions of the plane.

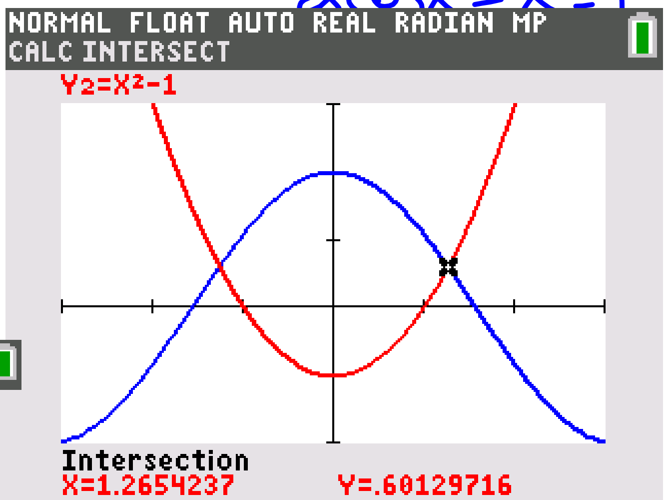
EXAMPLE 3 Using a Calculator

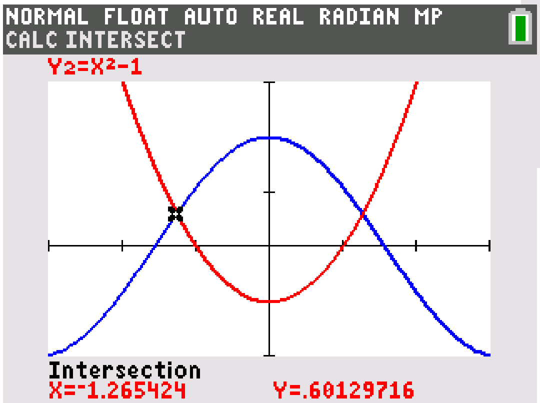
Find the area of the region enclosed by the graphs of $y = 2\cos x$ and $y = x^2 - 1$.

$2\cos x = x^2 - 1$



[–3, 3] by [–2, 3]





NORMAL FLOAT AUTO REAL RADIAN MP

1.2654237→J

1.2654237

$\int_{-J}^{J}(Y_1-Y_2)dX$

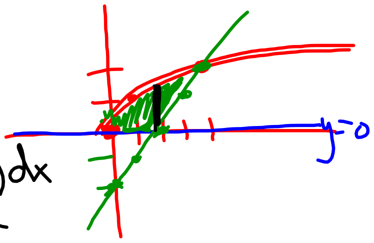
4.994907788

$$\int_{-1.265...}^{1.265...} 2\cos x-(x^2-1)\,dx$$

$$\boxed{4.995\ u^2}$$

EXAMPLE 4 Finding Area Using Subregions

Find the area of the region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.



$$\int_0^2 \sqrt{x}\,dx + \int_2^4 \sqrt{x}-(x-2)\,dx$$

$x^{1/2}$ $\qquad$ $\sqrt{x}-x+2$

$$\frac{2x^{3/2}}{3}\Big|_0^2 \qquad \frac{2x^{3/2}}{3}-\frac{x^2}{2}+2x\Big|_2^4$$

$$\frac{2}{3}\sqrt{x^3}\Big|_0^2 \qquad \frac{2}{3}\sqrt{x^3}-\frac{x^2}{2}+2x\Big|_2^4$$

$$\left(\frac{4\sqrt{2}}{3}\right)-(0) + \left(\frac{16}{3}-\cancel{8}+\cancel{8}\right)-\left(\frac{4\sqrt{2}}{3}-2+4\right)$$

$$\cancel{\frac{4\sqrt{2}}{3}} + \frac{16}{3} - \cancel{\frac{4\sqrt{2}}{3}} - 2$$

$$\frac{16}{3}-\frac{6}{3}=\boxed{\frac{10}{3}u^2}$$

Option 2:

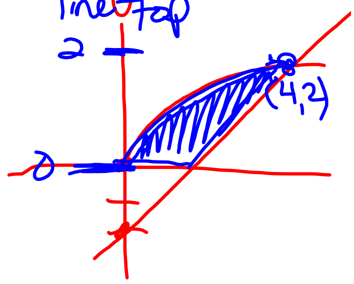
① solve equation as $x=$

$y=\sqrt{x}$ $\qquad$ $y=x-2$

$y^2=x$ $\qquad$ $y+2=x$

$x=y^2$ $\qquad$ $x=y+2$

bottom $\qquad$ line top



$$\int_0^2 y+2-y^2\,dy$$

$$\frac{y^2}{2}+2y-\frac{y^3}{3}\Big|_0^2$$

$$\left(2+4-\frac{8}{3}\right)-(0)$$

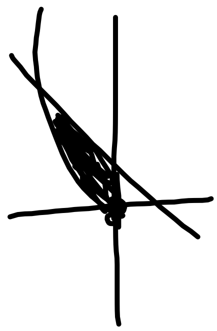
$$6-\frac{8}{3}$$

$$\frac{18}{3}-\frac{8}{3}$$

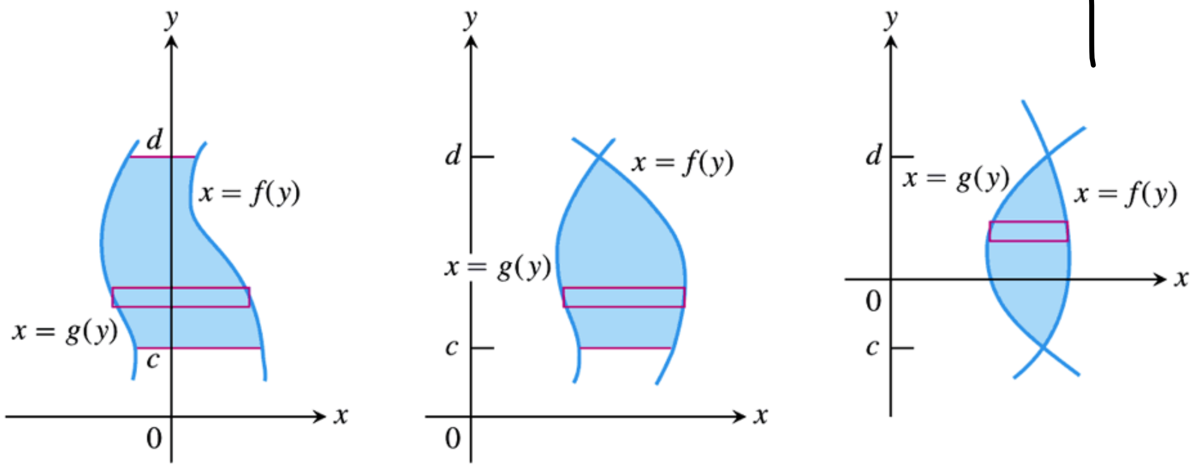
$$\boxed{10/3\ u^2}$$

Integrating with Respect to y

If the boundaries of a region are more easily described by functions of y, use horizontal approximating rectangles.



For regions like these



use this formula

$$A = \int_c^d [f(y) - g(y)]\,dy.$$

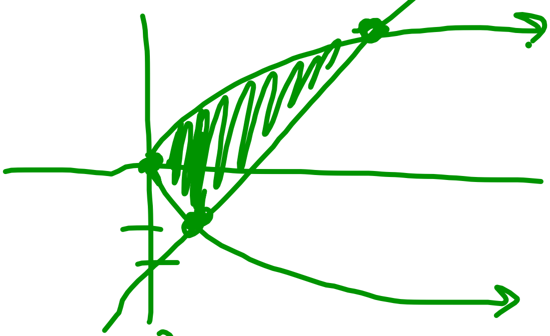
Find the area of the region enclosed by

$\sqrt{x} = \sqrt{y^2}$ and $x = y + 2$

$y = \pm\sqrt{x}$

$y = x - 2$

$$\int_{-1}^{2} y + 2 - y^2 \, dy$$



$\frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^{2}$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(2 + 4 - \frac{8}{3}) - (\underbrace{\frac{1}{2} - 2}_{} + \frac{1}{3})$

$(y-2)(y+1) = 0$

$6 - \frac{8}{3} + 1.5 - \frac{1}{3}$

$y = 2, -1$

$(\ , -1)(\ , 2)$

$7.5 \quad -3$

$\boxed{4.5 \, u^2}$

Homework 8.2:

Day 1: 3,6,9,15,18,21,30

Day 2: 12,24,27,36,39,42