

Section 8.1

Integral As Net Change

What you'll learn about



- Linear Motion Revisited
- General Strategy
- Consumption Over Time
- Net Change from Data
- Work

...and why

The integral is a tool that can be used to calculate net change and total accumulation.

Example Linear Motion Revisited

$v(t) = 10 - 2t$ is the velocity in m/sec of a particle moving along the x -axis when $0 \leq t \leq 9$. Use analytic methods to:

- (a) Determine when the particle is moving to the right, to the left, and stopped.
- (b) Find the particle's displacement for the given time interval.
- (c) If $s(0) = 3$, what is the particle's final position?
- (d) Find the total distance traveled by the particle.

EXAMPLE 4 Modeling the Effects of Acceleration

A car moving with initial velocity of 5 mph accelerates at the rate of $a(t) = 2.4t$ mph per second for 8 seconds.

(a) How fast is the car going when the 8 seconds are up?

(b) How far did the car travel during those 8 seconds?

$$\text{vel} = \int \text{accel}$$

$$v = \int_{0 \text{ sec}}^{8 \text{ sec}} 2.4t \, dt \frac{\text{mph}}{\text{sec}}$$

$$\frac{2.4t^2}{2} \Big|_0^8 \frac{\text{sec mph}}{\text{sec}}$$

$$v = 1.2t^2 \Big|_0^8 \text{ mph}$$

$$(1.2(8^2)) - (1.2(0^2))$$

$$v = 76.8 \text{ mph}$$

$$\text{vel} = \underline{81.8 \text{ mph}}$$

b)

$$\text{pos} = \int \text{vel}$$

$$\text{accel} = 2.4t$$

$$\text{vel} = 1.2t^2 + 5$$

$$\text{pos} = \int_{0 \text{ Sec}}^{8 \text{ sec}} 1.2t^2 + 5 \, dt \frac{\text{mi}}{\text{hr}}$$

$$\frac{1.2t^3}{3} + 5t \Big|_0^8 \frac{\text{sec mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$.4t^3 + 5t \Big|_0^8$$

$$(.4(8)^3 + 5(8)) - (0)$$

$$\frac{244.8 \text{ mi}}{3600}$$

$$.068 \text{ mi}$$

$$\text{vel} = \int \text{accel}$$

$$\text{pos} = \int \text{vel}$$

directions = underline labels

Example Potato Consumption

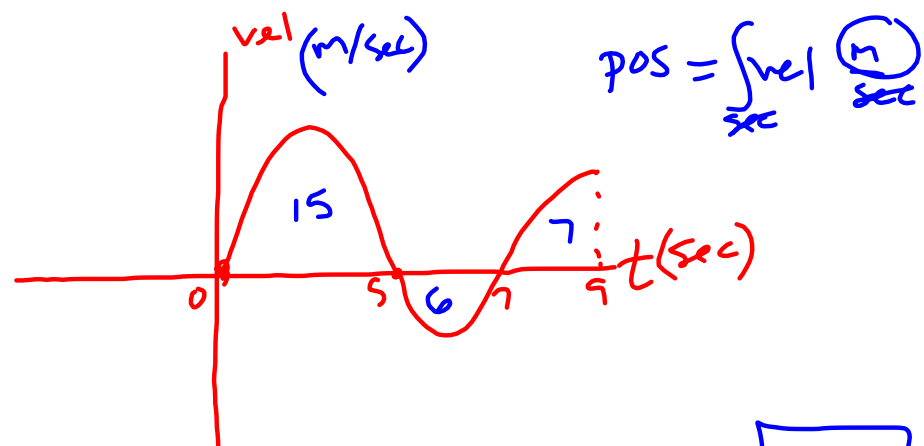


From 1970 to 1980, the rate of potato consumption in a particular country was $C(t) = 2.2 + 1.1^t$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1975?

Homework 8.1:

Day 1: 1-11 odd

Day 2: 12-17, 20-22, 25, 37

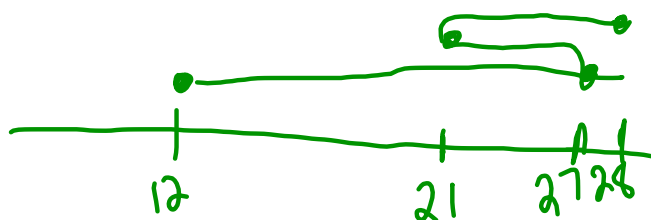


Displacement $15 - 6 + 7 = \boxed{16m}$

Total Distance $15 + 6 + 7 = \boxed{28m}$

Start on # line @ 12

Final Position = $12 + 16 = \boxed{28m} \checkmark$



EXAMPLE 6 Finding Gallons Pumped from Rate Data

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in Table 8.1. How many gallons were pumped during that hour?

TABLE 8.1 Pumping Rates

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

60 min

$$\int_0^{60} \text{vel} \frac{\text{gal}}{\text{min}}$$

~~min.gal~~
min

$$T = \frac{h}{2}$$

$$T = \frac{5}{2} (58 + 2(60) + 2(65) + 2(64) + 2(58) + 2(57) + 2(55) + 2(55) + 2(59) + 2(60) + 2(60) + 2(63))$$

$$T = \frac{5}{2} (1433)$$

$$\boxed{3582.5 \text{ gal}}$$