Section 8.1

Integral As Net Change

What you'll learn about

- Linear Motion Revisited
- General Strategy
- Consumption Over Time
- Net Change from Data
- Work

$s(t)$ - position

$s'(t) = v(t) =$ velocity

$s''(t) = v'(t) = a(t) =$ acceleration

vel $= \int_{0\,sec}^{8\,sec}$ accel $\frac{m}{s^2} \cdot s = \frac{m}{s}$

pos $= \int_{0\,sec}$ vel $\frac{m}{sec} = sec \cdot \frac{m}{sec}$

...and why

The integral is a tool that can be used to calculate net change and total accumulation.

Example Linear Motion Revisited

$\boxed{v(t) = 10 - 2t}$ is the velocity in $\boxed{\text{m/sec}}$ of a particle moving along the x-axis when $\boxed{0 \le t \le 9.}$ Use analytic methods to:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval.

(c) If $s(0) = 3$, what is the particle's final position? $\boxed{12m}$

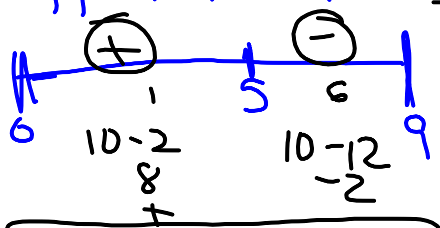
(d) Find the total distance traveled by the particle.

t=6 starting at 3

a) stopped ⇒ vel = 0 $\boxed{10-2t} = 0$

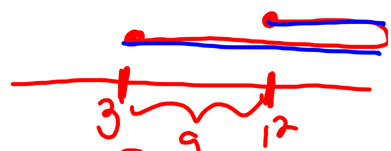
$10 = 2t$

$5 = t$ sec



$\boxed{\begin{array}{ll} \text{Rt} & 0 \le t < 5 \\ \text{left} & 5 < t \le 9 \end{array}}$

b) displacement



disp = $\int$ vel

$$\underbrace{\int_0^{5} 10-2t\,dt}_{\text{Rt}} + \underbrace{\left|\int_5^9 10-2t\,dt\right|}_{\text{left}}$$

5 sec

$$10t - t^2\Big|_0^5 + \left|10t - t^2\Big|_5^9\right|$$

$$(50-25) - (0) \qquad (90-81) - (50-25)$$

$$25\text{m} \qquad 9 - 25$$

$$|-16|$$

$25m + 16m$

$\boxed{41m}$

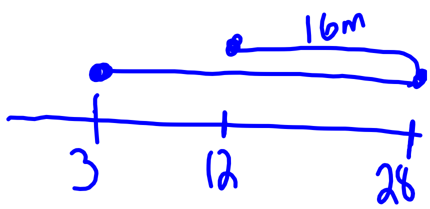
disp = $\int_0^9 10 - 2t\,dt$

$10t - t^2\Big|_0^9$

$(90-81) - (0)$

9m

*the object is 9 m to the right of where it started.



Example Potato Consumption

$\int$ $\boxed{f'}$ rate

From 1970 to 1980, the rate of potato consumption in a particular country was $\boxed{C(t) = 2.2 + 1.1^t}$ millions of bushels per year, with t being years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1975?

2

1976-1970
6

$$\int_{2\,yr}^{6\,yr} 2.2 + 1.1^t \, dt$$

$\frac{\text{mil of bushels}}{\text{yr}}$

alpha
window
4

or math 9

14.692 mil of bushels

EXAMPLE 4 **Modeling the Effects of Acceleration**

A car moving with initial $\boxed{\text{velocity of 5 mph}}$ accelerates at the rate of $\boxed{a(t)} = 2.4t$ mph per second for 8 seconds.

(a) How fast is the car going when the 8 seconds are up?

(b) How far did the car travel during those 8 seconds?

a) fast = vel = $\int$ accel

$$\text{vel} = \int_{0\text{ sec}}^{8\text{ sec}} 2.4t\,dt \;\frac{\text{mph}}{\text{sec}}$$

$$\cancel{\text{sec}} \frac{\text{mph}}{\cancel{\text{sec}}}$$

$$\text{vel} = \frac{2.4t^2}{2}\Big|_0^8 = 1.2t^2\Big|_0^8$$

$$(1.2(64)) - (1.2(0))$$

$$76.8 - 0$$

76.8 mph

$$5 + 76.8 = \boxed{81.8} \text{ mph}$$

b)

$$s(t) = \int \text{vel}$$

$$s(t) = \int_0^{8\text{ sec}} \underbrace{1.2t^2 + 5}_{\text{vel}}\,dt \;\frac{\text{mi}}{\text{hr}}$$

$$\frac{1.2t^3}{3} + 5t\Big|_0^8$$

$$.4t^3 + 5t\Big|_0^8$$

$$(.4(8)^3 + 5(8)) - (0)$$

$$244.8 \cdot \cancel{\text{sec}} \cdot \frac{\text{mi}}{\cancel{\text{hr}}} \cdot \frac{1\,\cancel{\text{hr}}}{60\,\cancel{\text{min}}} \cdot \frac{1\,\cancel{\text{min}}}{60\,\cancel{\text{sec}}}$$

$$\frac{244.8\text{ mi}}{3600}$$

.068 mi

EXAMPLE 6 Finding Gallons Pumped from Rate Data

A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator to operate other machinery. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for one hour as shown in Table 8.1. How many gallons were pumped during that hour?

TABLE 8.1 Pumping Rates

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

Homework 8.1:

Day 1: 1-11 odd

Day 2: 12-17, 20-22, 25, 37