Section 7.4

Homework Day 2:
21-30 by 3,37,42

Exponential Growth and Decay

What you'll learn about

- Separable Differential Equations
- Law of Exponential Change
- Continuously Compounded Interest
- Modeling Growth with Other Bases
- Newton's Law of Cooling

$y=y_0e^{kt}$
$A=Pe^{rt}$
$y=ab^x$

... and why
Understanding the differential equation $\frac{dy}{dx}=ky$
gives us new insight into exponential growth and decay.

6. $\frac{dy}{dx} = (\cos y)^2$

$\frac{1}{\cos^2 y}dy = 1dx$

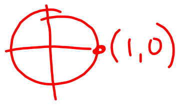
$x=0$
$y=0$

$\sec^2 y\,dy = 1dx$

$\tan y = x + C$

$\tan 0 = 0 + C$

$0 = C$



$\tan^{-1}\tan y = \tan^{-1} x$

$y = \tan^{-1} x$

3. $\frac{1}{y}\frac{dy}{dx} = \frac{\cancel{y}}{x}\frac{1}{\cancel{y}}$

$y=2$
$x=2$

$\frac{1}{y}dy = \frac{1}{x}dx$

$\ln y = \ln x + c$

$\ln 2 = \ln 2 + c$

$-\ln 2 \quad -\ln 2$

$c = 0$

$\cancel{\ln} y = \cancel{\ln} x$

$y = x$

EXAMPLE 4 Choosing a Base

At the beginning of the summer, the population of a hive of bald-faced hornets (which are actually wasps) is growing at a $\boxed{\text{rate proportional to the population}}$. From a population of 10 on May 1, the number of hornets grows to 50 in thirty days. If the growth continues to follow the same model, how many days after May 1 will the population reach 100?

$$y = y_0 e^{kt}$$

$$(0, 10)\ (30, 50)\ (t, 100)$$

$$y_0 = 10$$

$$50 = 10e^{k \cdot 30}$$

$$5 = e^{30k}$$

$$\log_e 5 = 30k$$

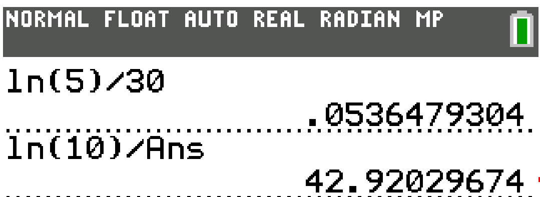
$$\frac{\ln 5}{30} = \frac{30k}{30}$$

$$k = .053\ldots$$

$$\frac{100}{10} = \frac{10e^{.053\ldots t}}{10}$$

$$10 = e^{.053\ldots t}$$

$$\frac{\ln 10}{.053\ldots} = \frac{.053\ldots t}{.053\ldots}$$



$$t = 42.920 \text{ days}$$

Example Finding Half-Life

Find the half-life of a radioactive substance with decay equation $y = y_0 e^{-kt}$.

$$y = y_0 e^{-kt}$$

$$\frac{\frac{1}{2}\cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} e^{-kt}}{\cancel{y_0}}$$

$$\frac{1}{2} = e^{-kt}$$

$$\frac{1}{2}^{-1}$$

$$\frac{\ln\frac{1}{2}}{-k} = \frac{-kt}{-k}$$

$$t = \frac{-\ln\frac{1}{2}}{k}$$

$$t = \frac{\ln 2}{k} \qquad k > 0$$

Ex: Find the half-life of:

$$\frac{dy}{dt} = \underline{k}y$$

$$\frac{dy}{dt} = -.235y$$

$$t = \frac{\ln 2}{.235}$$

$$t = 2.950$$

Ex: Scientists who use carbon-14 dating use 5700 years for its half-life. Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

$$y = a b^{t}$$

$$.9Y_0 = Y_0\left(\tfrac{1}{2}\right)^{t/5700}$$

$$.9 = .5^{t/5700}$$

$$5700 \log_{.5} .9 = \frac{t}{5700} \quad 5700$$

$$t = 866 \text{yrs}$$

Half-life

The **half - life** of a radioactive substance with rate constant k $(k > 0)$ is

$$\text{half-life} = \frac{\ln 2}{k}.$$

$$T - T_S = (T_O - T_S)e^{-kt},$$

Where T_O is the temperature at time $t = 0$.

EXAMPLE 6 Using Newton's Law of Cooling

A hard-boiled egg at 98°C is put in a pan under running 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. How much longer will it take the egg to reach 20°C?