

Section 7.1

Slope Fields and
Euler's Method

Homework:

Day 1: 3-24 by 3

Day 2: 27-48 by 3

What you'll learn about

- Differential Equations
- Slope Fields
- Euler's Method

... and why

Differential equations have been a prime motivation for the study of calculus and remain so to this day.

Differential Equation

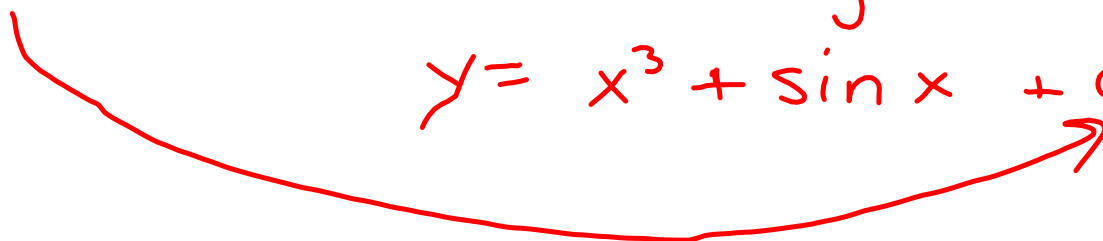
An equation involving a derivative is called a **differential equation**. The **order of a differential equation** is the order of the highest derivative involved in the equation.

Example Solving a Differential Equation

Find all functions y that satisfy $\frac{dy}{dx} = 3x^2 + \cos x$.

$$y = x^3 + \sin x + C$$

$\int \# \rightarrow$ no C
 $\int \# \rightarrow + C$



EXAMPLE 1 Solving a Differential Equation

Find all functions y that satisfy $dy/dx = \sec^2 x + 2x + 5$.

~~SS~~

$$y = \tan x + x^2 + 5x + C$$

First-order Differential Equation

If the general solution to a first-order differential equation is continuous, the only additional information needed to find a unique solution is the value of the function at a single point, called an initial condition. A differential equation with an initial condition is called an **initial-value problem**. It has a unique solution, called the particular solution to the differential equation.

Handwritten notes:
- A red circle around "general" has an arrow pointing to "all!".
- A red box around "unique" has "specific" written above it.
- A red box around "initial condition".
- A red box around "particular solution".
- A red underline under "single point".

Example Solving an Initial Value Problem



Find the particular solution to the equation $\frac{dy}{dx} = e^{2x} - 3x$ whose graph

passes through the point $\left(1, \frac{1}{2}\right)$.

$$y = \frac{e^{2x}}{2} - \frac{3x^2}{2} + C$$

$$\frac{1}{2} = \frac{e^2}{2} - \frac{3}{2} + C$$

$$2 = e^2 + C$$

$$C = 2 - \frac{e^2}{2}$$

$$y = \frac{e^{2x}}{2} - \frac{3x^2}{2} + 2 - \frac{e^2}{2}$$

Example Using the Fundamental Theorem to Solve an Initial Value Problem



Find the solution to the differential equation

$$f'(x) = \cos(x^2) \text{ for which } f(3) = 5.$$

$f(x) = y$ $y = \int_3^x \cos(t^2) dt + 5$

EXAMPLE 4 Using the Fundamental Theorem to Solve an Initial Value Problem

Find the solution to the differential equation $f'(x) = e^{-x^2}$ for which $f(7) = 3$.

$$y = \int_7^x e^{-t^2} dt + 3$$