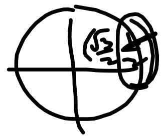
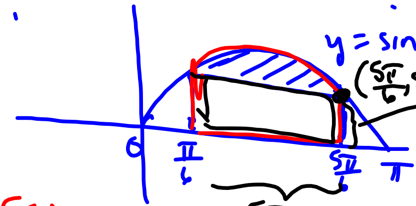


48.

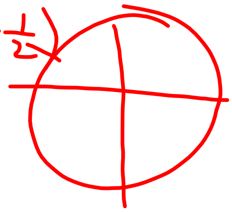


$\left(\frac{5\pi}{6}, \sin\frac{5\pi}{6}\right) \to w = \sin\frac{\pi}{6} = \frac{1}{2}$

$\frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3} = l$

$$\int_{\pi/6}^{5\pi/6} \sin x \, dx - \frac{2\pi}{3} \cdot \frac{1}{2}$$

$$-\cos x \Big|_{\pi/6}^{5\pi/6} - \frac{\pi}{3}$$



$$\left(-\cos\frac{5\pi}{6}\right) + \left(+\cos\frac{\pi}{6}\right)$$

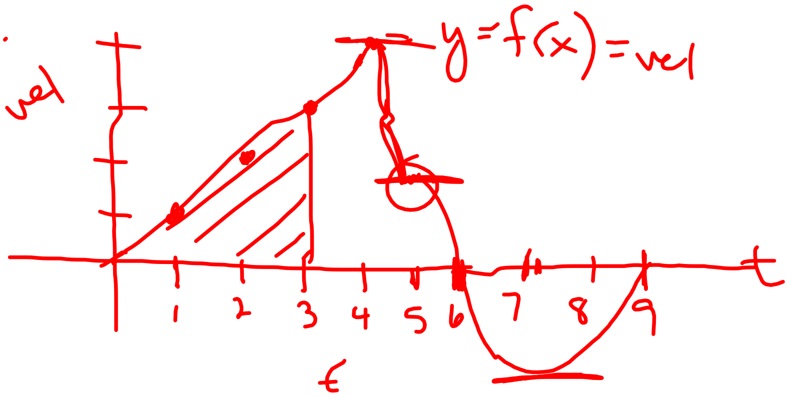
$$-\left(-\frac{\sqrt{3}}{2}\right)$$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{2}}$$

$$\sqrt{3} - \frac{\pi}{3}$$

58.



$$S = \int_0^t f(x)\,dx$$

$$S(3) = \int_0^3 f(x)\,dx = \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$$

Section 6.5

Homework: 1,4,7,9,10a,19ab

Trapezoidal Rule

What you'll learn about

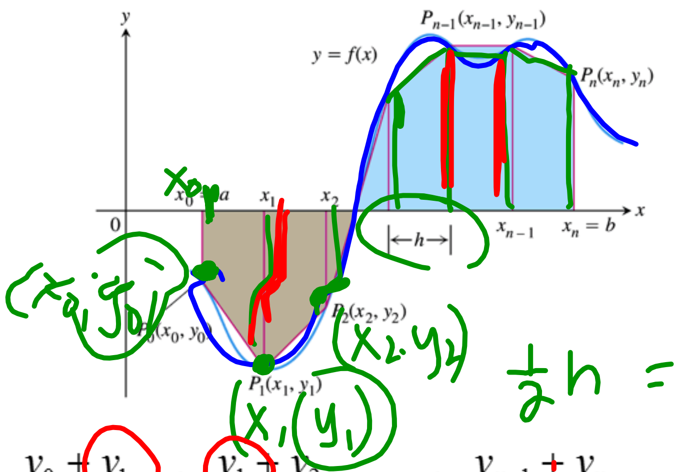
$T = \frac{1}{2}h(b_1 + b_2)$

- Trapezoidal Approximations
- Other Algorithms Simpson's Rule
- Error Analysis

… and why

Some definite integrals are best found by numerical approximations, and rectangles are not always the most efficient figures to use.

Trapezoidal Approximations



$\frac{1}{2}h = \frac{h}{2} = h\left(\frac{\quad}{2}\right)$

$$\int_a^b f(x)dx \approx h \cdot \frac{y_0 + y_1}{2} + h \cdot \frac{y_1 + y_2}{2} + \ldots + h \cdot \frac{y_{n-1} + y_n}{2}$$

$$= h\left(\frac{y_0}{2} + y_1 + y_2 + \ldots + y_{n-1} + \frac{y_n}{2}\right)$$

$$\boxed{= \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n),}$$

where $y_0 = f(a), \quad y_1 = f(x_1), \ \ldots, \ y_{n-1} = f(x_{n-1}), \ y_n = f(b).$

The Trapezoidal Rule

To approximate $\int_a^b f(x)dx$, use

$$\boxed{T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n),}$$

where $[a, b]$ is partitioned into n subintervals of equal length

$$\boxed{h = (b-a)/n.}$$

Equivalently, $\boxed{T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2},}$

where LRAM_n and RRAM_n are the Rienamm sums using the left and right endpoints, respectively, for f for the partition.

$$\int_5^{10} f(x)\,dx$$

n= 10 trap

$$h = \frac{10-5}{10} = \frac{5}{10} = \frac{1}{2}$$

1. The function f is continuous on the closed interval [1,7] and has values that are given below:

x	1	4	6	7
$f(x)$	10	30	40	20

Using the subintervals [1,4], [4,6], and [6,7], what is the trapezoidal approximation of $\int_1^7 f(x)\,dx$?

$\frac{1}{2}h$ $\frac{1}{2}\cdot h$

$$\frac{3}{2}(10+30) + \frac{2}{2}(30+40) + \frac{1}{2}(40+20)$$

$$\frac{3}{\cancel{2}}(\overset{20}{\cancel{40}}) \qquad 1(70) \qquad \frac{1}{2}(60)$$

$$60 + 70 + 30$$

$$\boxed{160}$$

EXAMPLE 2 Averaging Temperatures

An observer measures the outside temperature every hour from noon until midnight, recording the temperatures in the following table.

Time	N	1	2	3	4	5	6	7	8	9	10	11	M
	0												12
Temp	63	65	66	68	70	69	68	68	65	64	62	58	55

$\frac{1}{b-a}$

What was the $\boxed{\text{average}}$ temperature for the 12-hour period? $h=1$

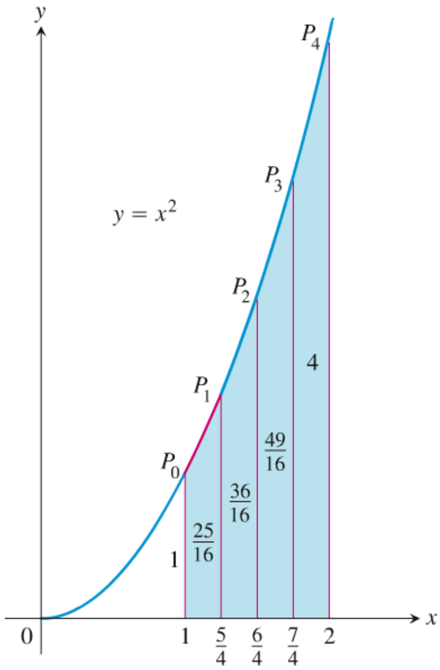
$$T=\frac{1}{2}\left(63+2(65)+2(66)+2(68)+2(70)+2(69)+2(68)+2(68)+2(65)+2(64)+2(62)+2(58)+55\right)$$

$$\frac{1}{2}(1564)$$

$$\frac{1}{12-0}(782)=\frac{782}{12}=65.167^\circ$$

EXAMPLE 1 Applying the Trapezoidal Rule

Use the Trapezoidal Rule with $n = 4$ to estimate $\int_1^2 x^2\,dx$. Compare the estimate with the value of NINT $(x^2, x, 1, 2)$ and with the exact value.



$h = \frac{2-1}{4} = \frac{1}{4}$ $\quad \frac{b-a}{n}$

$y = x^2$

$T = \frac{h}{2}$

$\frac{1}{2} \cdot h$

$\frac{\frac{1}{4}}{2}$ or $\frac{1}{2} \cdot \frac{1}{4}$

TABLE 6.4

x	$y = x^2$
1	1
$\frac{5}{4}$	$\frac{25}{16}$
$\frac{6}{4}$	$\frac{36}{16}$
$\frac{7}{4}$	$\frac{49}{16}$
2	4

$T = \frac{1}{8}\left(1 + 2\left(\frac{25}{16}\right) + 2\left(\frac{36}{16}\right) + 2\left(\frac{49}{16}\right) + 4\right)$

$T = \frac{1}{8}\left(\underset{\frac{16}{16}}{1} + \frac{50}{16} + \frac{72}{16} + \frac{98}{16} + \underset{\frac{64}{16}}{4}\right)$

$\frac{1}{8}\left(\frac{\cancel{300}^{75}}{\cancel{16}_{4}}\right) = \frac{75}{32} = 2.34375$

$\int_1^2 x^2\,dx = 2.\overline{3}$

Simpson's Rule

To approximate $\int_a^b f(x)dx$, use

$$S = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right),$$

where $[a, b]$ is partitioned into an even number n subintervals of equal length $h = (b-a)/n$.

EXAMPLE 3 Applying Simpson's Rule

Use Simpson's Rule with $n = 4$ to approximate $\int_0^2 5x^4\,dx$.

TABLE 6.5

x	$y = 5x^4$
0	0
$\frac{1}{2}$	$\frac{5}{16}$
1	5
$\frac{3}{2}$	$\frac{405}{16}$
2	80