6.4 Fundamental Theorem of Calculus

What you will learn about . . .

- Fundamental Theorem, Part 1
- Graphing the Function $\int_a^x f(t)\,dt$
- Fundamental Theorem, Part 2
- Area Connection
- Analyzing Antiderivatives Graphically

and why . . .

The Fundamental Theorem of Calculus is a triumph of mathematical discovery and the key to solving many problems.

THEOREM 4 (continued) The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$F\Big|_a^b = F(b) - F(a)$$

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem.**

EXAMPLE 5 Evaluating an Integral

Evaluate $\int_{-1}^{3}(x^3 + 1)\,dx$ using an antiderivative.

$$\frac{x^4}{4} + x \Big|_{-1}^{3}$$

$-1^4 = -1$

$(-1)^4 = 1$

$$\left(\frac{81}{4} + 3\right) - \left(\frac{1}{4} + -1\right)$$

$$\frac{81}{4} + \frac{12}{4}$$

$$\left(\frac{93}{4}\right) - \left(-\frac{3}{4}\right)$$

$$\frac{96}{4} = \boxed{24}$$

EXAMPLE 6 **Finding Area Using Antiderivatives**

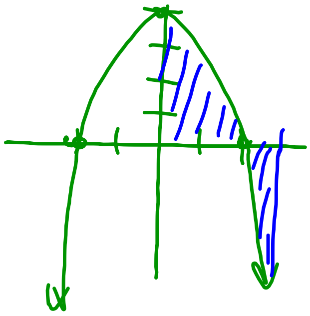
Find the area of the region between the curve $y = 4 - x^2$, $0 \le x \le 3$, and the x-axis.

$$\int_0^3 4 - x^2\,dx$$

$$4x - \frac{x^3}{3}\Big|_0^3$$

$$(12 - 9) - (0)$$

evaluate Integral $\boxed{3}$



$$\int_0^2 4 - x^2\,dx + \left|\int_2^3 4 - x^2\,dx\right|$$

subtract + +

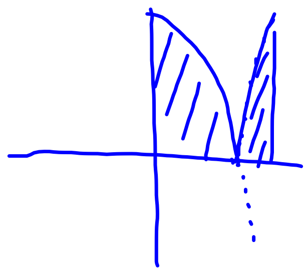
$$4x - \frac{x^3}{3}\Big|_0^2 + \left|4x - \frac{x^3}{3}\Big|_2^3\right|$$

$$\left(8 - \frac{8}{3}\right) - (0) \qquad (12 - 9) - \left(\frac{16}{3}\right)$$

$$\frac{24}{3} - \frac{8}{3} \qquad \frac{9}{3} - \frac{16}{3}$$

$$\frac{9}{3} - \frac{16}{3}$$

$$\frac{16}{3} + \left|-\frac{7}{3}\right|$$

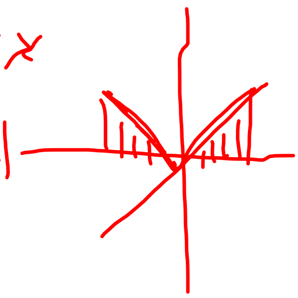


$$\frac{16}{3} + \frac{7}{3}$$

$$\frac{23}{3} = 7\tfrac{2}{3}$$

$y = x$

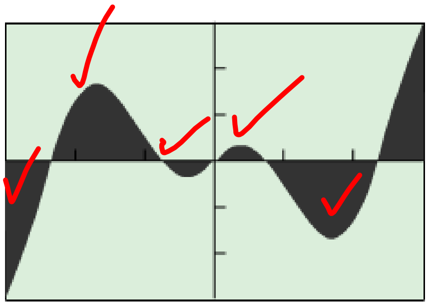
$y = |x|$



EXAMPLE 7 **Finding Area Using NINT**

Find the area of the region between the curve $y = x \cos 2x$ and the x-axis over the interval $-3 \le x \le 3$ (Figure 6.29).

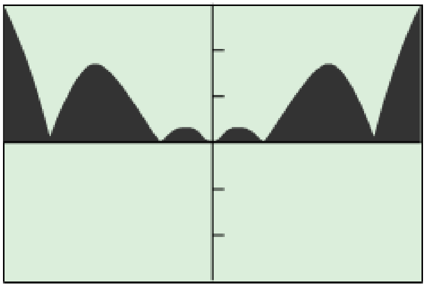
$$\int_{-3}^{3} x \cos 2x \, dx = \boxed{0}$$



[−3, 3] by [−3, 3]

(a)

$$\int_{-3}^{3} \left| x \cos(2x) \right| dx = \boxed{5.425}$$



[−3, 3] by [−3, 3]

(b)

NORMAL FLOAT AUTO REAL RADIAN MP

$\int_{-3}^{3}(|X\cos(2X)|)dX$

5.425029484

EXAMPLE 8 Using the Graph of f to Analyze $h(x) = \int_a^x f(t)\,dt$

The graph of a continuous function f with domain $[0, 8]$ is shown in Figure 6.30. Let h be the function defined by $\boxed{h(x) = \int_1^x f(t)\,dt.}$ (0, 8)

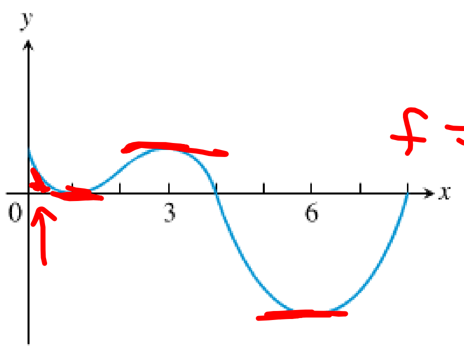
(a) Find $h(1)$.

(b) Is $h(0)$ positive or negative? Justify your answer.

(c) Find the value of x for which $h(x)$ is a maximum.

(d) Find the value of x for which $h(x)$ is a minimum.

(e) Find the x-coordinates of all points of inflections of the graph of $y = h(x)$. 1



$f = h'$

a) Find $h(1) = \int_1^1 f(t)\,dt = \boxed{0}$

b) $h(0) = \int_1^0 f(t)\,dt$

$= -\underbrace{\int_0^1 f(t)\,dt}_{+}$

$-$ $+$

$-$

c) $h(x)$ is a maximum $(+-)$

h': $+$ | $+$ | $-$ with marks at 1, 4, 8

when $x = 4$

d) min $(-+)$
none

e) Infl $x = 1, x = 3, x = 6$

Homework 6.4 Day 2: 27-51 by 3, 58,64