6.4 Fundamental Theorem of Calculus

What you will learn about . . .

- Fundamental Theorem, Part 1
- Graphing the Function $\int_a^x f(t)\,dt$
- Fundamental Theorem, Part 2
- Area Connection
- Analyzing Antiderivatives Graphically

and why . . .

The Fundamental Theorem of Calculus is a triumph of mathematical discovery and the key to solving many problems.

THEOREM 4 The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x). \quad (1)$$

*a more indepth proof is on pages 298.

F(x)=original

f(t)=derivative of F(x)

EXAMPLE 1 Applying the Fundamental Theorem

Find

$$\cancel{\frac{d}{dx}\int_{-\pi}^{x}} \cos t \,\cancel{dt}$$

by using the Fundamental Theorem.

$\boxed{\cos x}$

$$\frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\,dt$$

EXAMPLE 2 **The Fundamental Theorem with the Chain Rule**

Find dy/dx if $y = \int_1^{x^2} \cos t \, dt$.

$$\cancel{\frac{d}{dx}\int_1^{x^2}} \cos t \, \cancel{dt}$$

$$\cos(x^2) \cdot 2x$$

$$\boxed{2x\cos(x^2)}$$

EXAMPLE 3 **Variable Lower Limits of Integration**

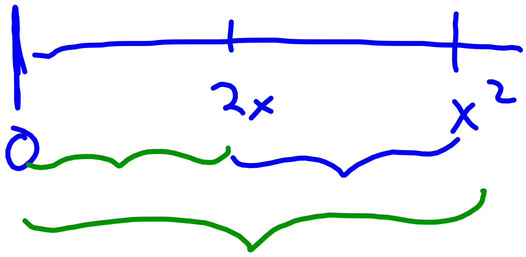
Find dy/dx

(a) $y = \int_x^5 3t \sin t \, dt$

$\cancel{\frac{d}{dx}} - \cancel{\int_5^x} 3t \sin t \, \cancel{dt}$

$\boxed{-3x\sin x}$

Find dy/dx: (b) $y = \int_{2x}^{x^2} \frac{1}{2+e^t}\,dt$



$$\cancel{\frac{d}{dx}\int_{0}^{x^2}} \frac{1}{2+e^{t}}\,\cancel{dt} - \cancel{\frac{d}{dx}\int_{0}^{2x}} \frac{1}{2+e^t}\,\cancel{dt}$$

$$\frac{1}{2+e^{x^2}}\cdot 2x - \frac{1}{2+e^{2x}}\cdot 2$$

$$\boxed{\frac{2x}{2+e^{x^2}} - \frac{2}{2+e^{2x}}}$$

EXAMPLE 4 **Constructing a Function with a Given Derivative and Value**

Find a function $y = f(x)$ with derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $\boxed{f(3) = 5.}$

$f(x) = y$

$\tan x$

$$y = \underbrace{\int_3^x \tan t \, dt + 5}_{0 + 5}$$

$y = 5$

Example: Construct a function with $dy/dx=e^x$ satisfying the condition that $f(4)= -7$.

$$y=\int_4^x e^t\,dt-7$$

Example: #10 Find dy/dx if: $\frac{dy}{dx} = \frac{d}{dx}\int_{6}^{x^2} \cot(3t)\,dt$

$\cot(3x^2) \cdot 2x$

$\frac{dy}{dx} = 2x\cot(3x^2)$

Example: #16 Find dy/dx if:

$$y = \int_{5x^2}^{25} \frac{t^2 - 2t + 9}{t^3 + 6}\, dt$$

$$\frac{d}{dx} y \;\cancel{=}\; \cancel{\frac{d}{dx}}\; -\int_{25}^{5x^2} \frac{t^2-2t+9}{t^3+6}\,\cancel{dt}$$

$$\frac{dy}{dx} = -\;\frac{(5x^2)^2 - 2(5x^2) + 9}{(5x^2)^3 + 6} \cdot 10x$$

$$-\frac{10x}{1} \cdot \frac{25x^4 - 10x^2 + 9}{125x^6 + 6}$$

$$\frac{-10x(25x^4 - 10x^2 + 9)}{125x^6 + 6}$$

$$\boxed{\frac{-250x^5 + 100x^3 - 90x}{125x^6 + 6}}$$

Homework 6.4 Day 1: 3-24 by 3