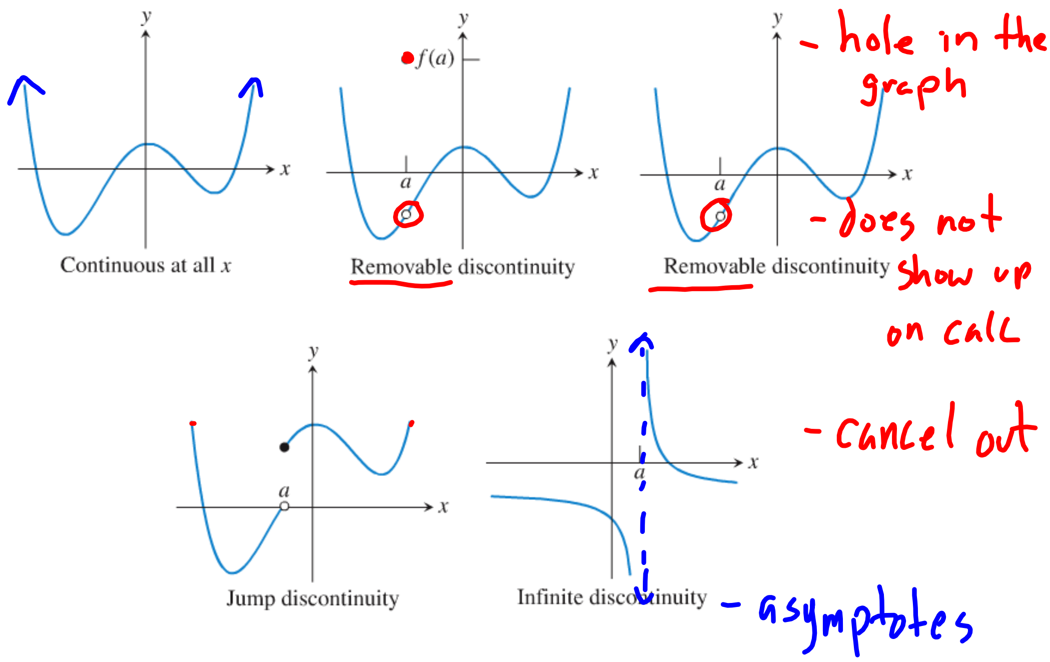
Section 1-2: Day 2

Functions & Their Properties

- Students will be able to represent functions numerically, algebraically and graphically
- Students will be able to determine the domain and range for the function
- Students will be able to analyze the function's characteristics such as extreme values, symmetry, asymptotes & end behavior

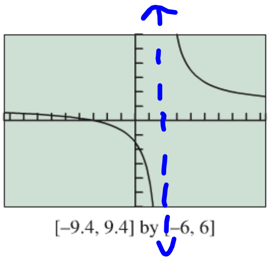
Continuity

- Graphically speaking, a function is continuous at a point if the graph does not come apart at that point.



Identifying Points of Discontinuity

Judging from the graphs, which of the following figures shows functions that are discontinuous at x = 2? Are any of the discontinuities removable?

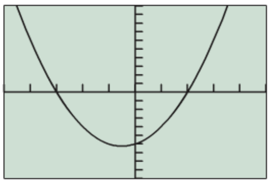


[–9.4, 9.4] by [–6, 6]

FIGURE 1.16 $f(x) = \dfrac{x+3}{x-2}$

Y, N

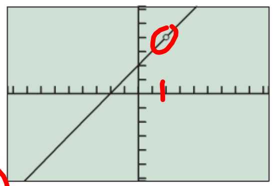
D: $(-\infty, 2) \cup (2, \infty)$



[–5, 5] by [–10, 10]

FIGURE 1.17 $g(x) = (x + 3)(x - 2)$

N, N



[–9.4, 9.4] by [–6.2, 6.2]

FIGURE 1.18 $h(x) = \dfrac{x^2 - 4}{x - 2}$

Y, Y

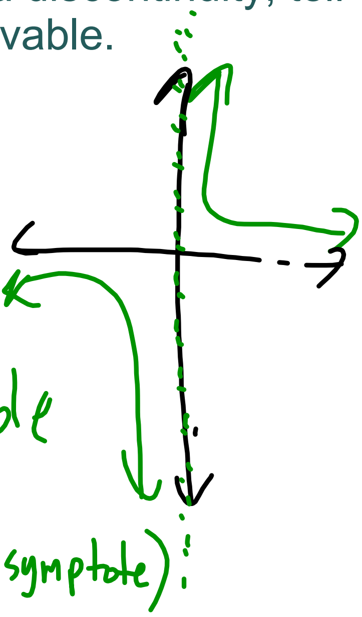
$h(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

cancels out
hole in graph

#21

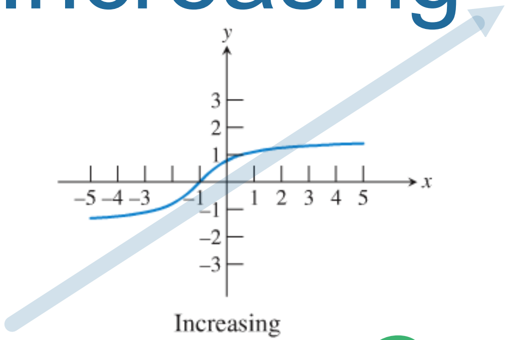
21. Graph the function and tell whether or not it has a point of discontinuity at x = 0. If there is a discontinuity, tell whether it is removable or nonremovable.

$$g(x) = \frac{3}{x}$$



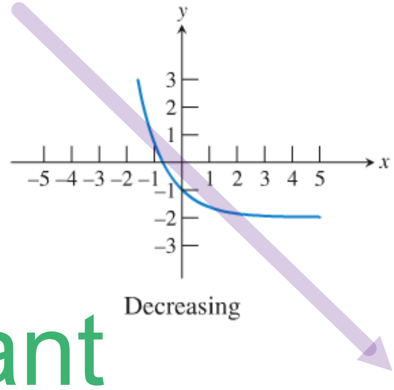
discontinuous
nonremovable
infinite (asymptote)

Increasing



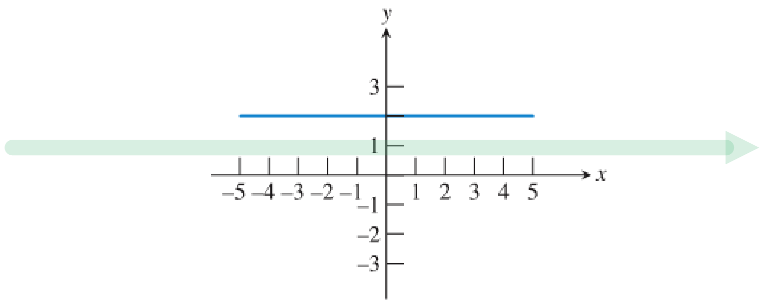
Increasing

Decreasing



Decreasing

Constant



Constant

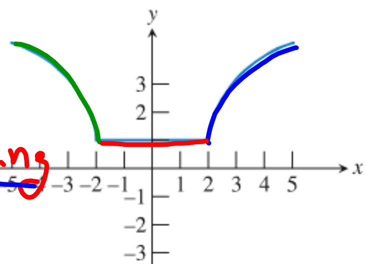
DEFINITION Increasing, Decreasing, and Constant Function on an Interval

A function f is **increasing** on an interval if, for any two points in the interval, a positive change in x results in a positive change in $f(x)$. as x gets larger y gets larger

A function f is **decreasing** on an interval if, for any two points in the interval, a positive change in x results in a negative change in $f(x)$. as x gets larger y gets smaller

A function f is **constant** on an interval if, for any two points in the interval, a positive change in x results in a zero change in $f(x)$. as x gets larger y stays the same

Use $\boxed{\text{x-values}}$ for intervals of increasing and decreasing

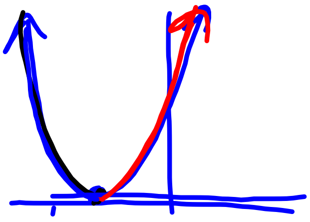


Decreasing on (−∞, −2]
Constant on [−2, 2]
Increasing on [2, ∞)

Analyzing a Function for Increasing-Decreasing Behavior

For each function, tell the intervals on which it is increasing and the intervals on which it is decreasing.

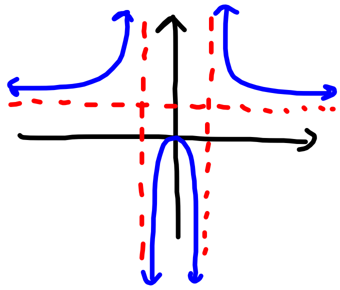
$$f(x)=(x+2)^2$$



INC $(-2,\infty)$

DEC $(-\infty,-2)$

$$g(x)=\frac{x^2}{x^2-1}$$



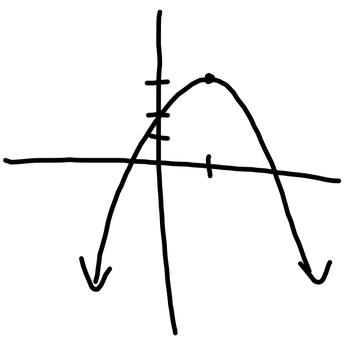
INC $(-\infty,-1)\cup(-1,0)$

DEC $(0,1)\cup(1,\infty)$

#33

33. Graph the function and identify intervals on which the function is increasing, decreasing or constant.

$$g(x)=3-(x-1)^2$$



INC $(-\infty,1)$

DEC $(1,\infty)$

D: $(-\infty,\infty)$

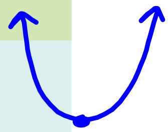
R: $(-\infty,3]$

DEFINITION Lower Bound, Upper Bound, and Bounded

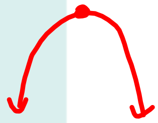
A function f is **bounded below** if there is some number b that is less than or equal to every number in the range of f. Any such number b is called a **lower bound** of f.

A function f is **bounded above** if there is some number B that is greater than or equal to every number in the range of f. Any such number B is called an **upper bound** of f.

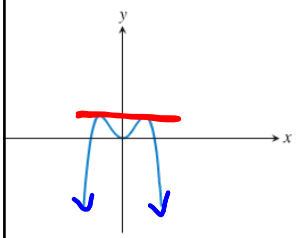
A function f is **bounded** if it is bounded both above and below.



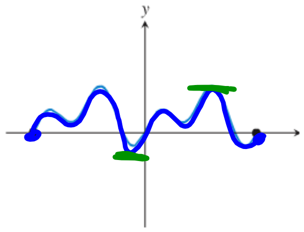
bounded below



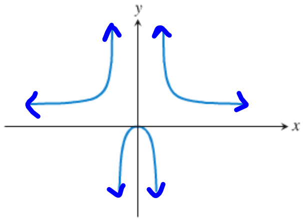
bounded above



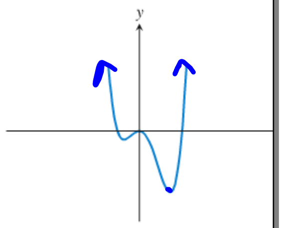
Bounded above
Not bounded below



Bounded



Not bounded above
Not bounded below

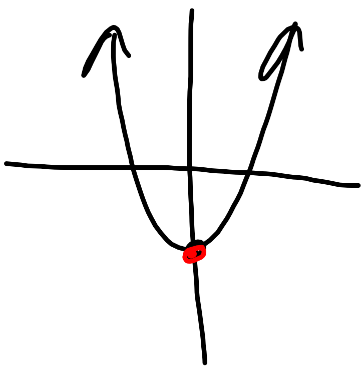


Not bounded above
Bounded below

Checking Boundedness

Identify each of these functions as bounded below, bounded above or bounded.

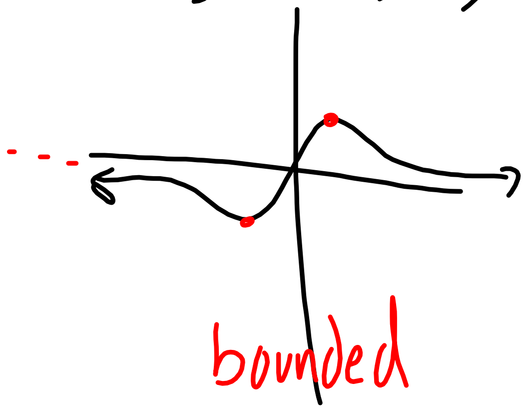
$$w(x) = 3x^2 - 4$$



lower bound

$$p(x) = \frac{x}{x^2 + 1}$$

D: $(-\infty, \infty)$

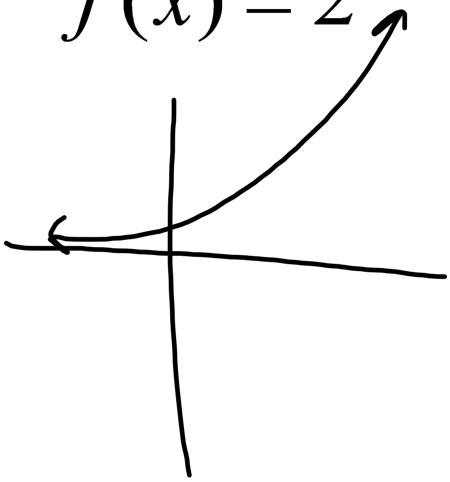


bounded

#33

37. Determine whether the function is bounded above, bounded below or bounded.

$$f(x) = 2^x$$



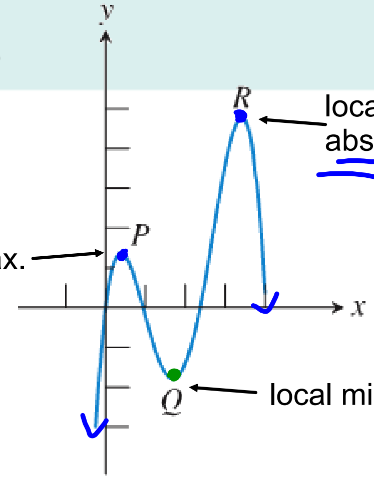
Bounded below

DEFINITION Local and Absolute Extrema

A **local maximum** of a function f is a value $f(c)$ that is greater than or equal to all range values of f on some open interval containing c. If $f(c)$ is greater than or equal to all range values of f, then $f(c)$ is the **maximum** (or **absolute maximum**) value of f.

A **local minimum** of a function f is a value $f(c)$ that is less than or equal to all range values of f on some open interval containing c. If $f(c)$ is less than or equal to all range values of f, then $f(c)$ is the **minimum** (or **absolute minimum**) value of f.

Local extrema are also called **relative extrema**.



Identifying Local Extrema

Decide whether $f(x) = x^4 - 7x^2 + 6x$ has any local maxima or local minima. If so, find each local maximum value or minimum value and the value of x at which each occurs.

Local Max $\boxed{y = 1.32 \ @ \ x = .46}$

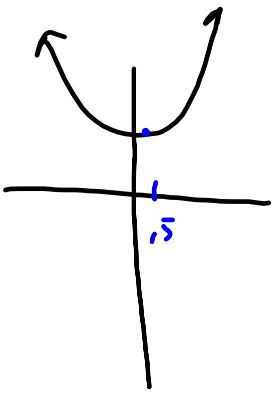
Local Min $y = -1.77 \ @ \ x = 1.60$

~~(.46, 1.32)~~

Local Min
Abs
$y = -24.06 \ @ \ x = -2.06$

#41

41. Use a grapher to find all local maxima and minima and the values of x where they occur. Give values rounded to two decimal places.



$f(x) = 4 - x + x^2$

local min $3.75 \ @ \ x = .5$

no local max

Symmetry

with respect to the y-axis

Symmetry with respect to the y-axis

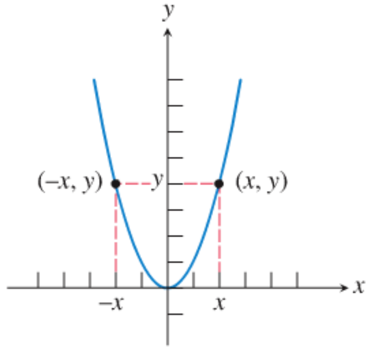
Example: $f(x) = x^2$

$f(x) = f(-x)$

$x^2 = (-x)^2$

$x^2 = x^2$

Graphically



Algebraically

For all x in the domain of f,

$$\boxed{f(-x) = f(x).}$$

Functions with this property (for example, x^n, n even) are **even** functions.

Numerically

x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

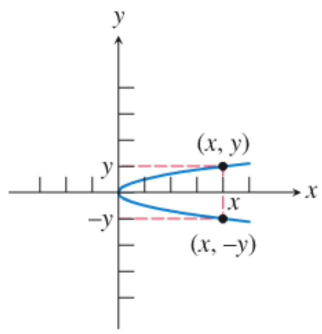
Symmetry

with respect to the x-axis

Symmetry with respect to the x-axis

Example: $x = y^2$

Graphically



Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Numerically

x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

Symmetry

with respect to the origin

Symmetry

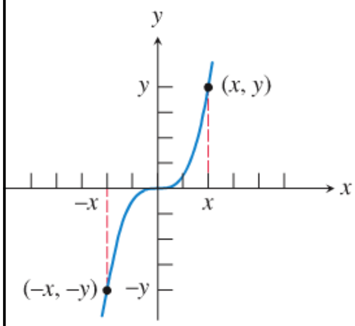
Symmetry with respect to the origin

Example: $f(x) = x^3$

$f(-x) = -f(x)$

$f(-x^3) = -x^3$

Graphically



Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

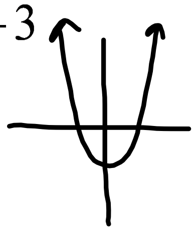
Numerically

x	y
-3	-27
-2	-8
-1	-1
1	1
2	8
3	27

Checking Functions for Symmetry

Tell whether each of the following functions is odd, even or neither.

$f(x) = x^2 - 3$



EVEN

$f(x) = f(-x)$

$= (-x)^2 - 3$

$x^2 - 3 = x^2 - 3$

$g(x) = x^2 - 2x - 2$

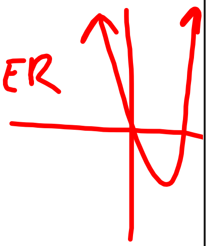
$g(x) = g(-x)$

$g(x) = (-x)^2 - 2(-x) - 2$

$= x^2 + 2x - 2$

$g(x) \neq g(-x)$

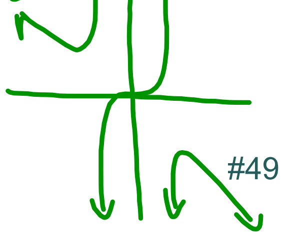
NEITHER



$h(x) = \dfrac{x^3}{4 - x^2}$

$h(x) = \dfrac{(-x)^3}{4-(-x)^2} = \dfrac{-x^3}{4-x^2}$

ODD



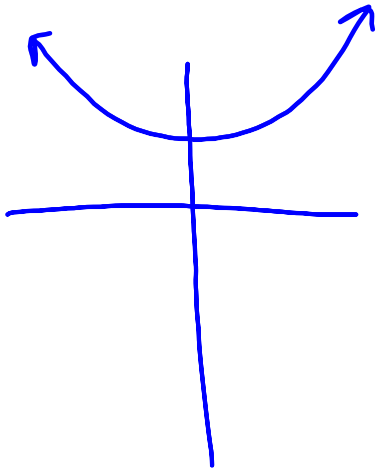
$h(-x) = -h(x)$

$= -\left(\dfrac{x^3}{4-x^2}\right)$

#49

49. State whether the function is odd, even or neither.

$$f(x) = \sqrt{x^2 + 2}$$



$f(-x) = \sqrt{(-x)^2 + 2}$

$f(-x) = \sqrt{x^2 + 2}$

$f(-x) = f(x)$

EVEN

DEFINITION Horizontal and Vertical Asymptotes

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$.

In limit notation:

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to +\infty} f(x) = b$$

The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

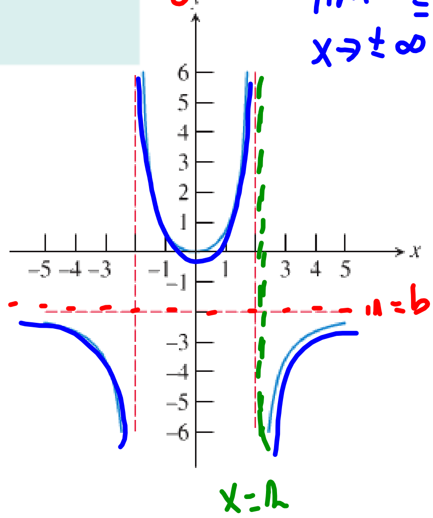
In limit notation:

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty$$

$$\boxed{\lim_{x \to \infty} f(x) = -2 \qquad \lim_{x \to -\infty} f(x) = -2}$$

$y = -2$ $\quad \lim_{x \to \pm\infty} = -2$

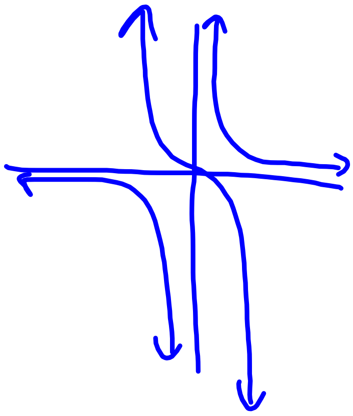
$$\boxed{\lim_{x \to 2^+} f(x) = -\infty \qquad \lim_{x \to 2^-} f(x) = \infty}$$



Identifying the Asymptotes of a Graph

Identify any horizontal or vertical asymptotes of the graph of

$$y = \frac{x}{x^2 - x - 2}$$



$f(-x) = -f(x)$

$= \frac{-x}{(-x)^2 - (-x) - 2}$ odd

$= \frac{-x}{x^2 + x - 2}$

#57

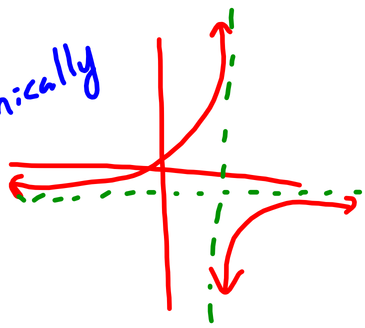
57. Use a method of your choice to find all horizontal and vertical asymptotes of the function.

$$f(x) = \frac{x+2}{3-x}$$

vert $x = 3$

horiz $y = -1$

Graphically



End Behavior

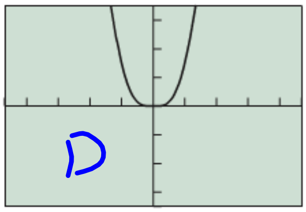
- how the function behaves as it goes off toward either "end" of the graph

A $y = \dfrac{3x}{x^2+1}$

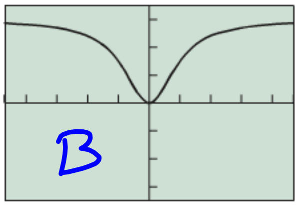
B $y = \dfrac{3x^2}{x^2+1}$

C $y = \dfrac{3x^3}{x^2+1}$

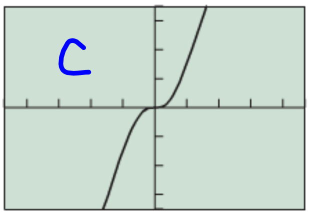
D $y = \dfrac{3x^4}{x^2+1}$



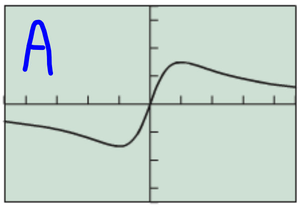
[–4.7, 4.7] by [–3.5, 3.5]
(i)



[–4.7, 4.7] by [–3.5, 3.5]
(iii)



[–4.7, 4.7] by [–3.5, 3.5]
(ii)



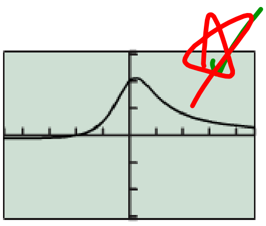
[–4.7, 4.7] by [–3.5, 3.5]
(iv)

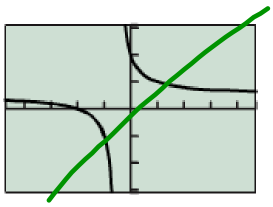
#65

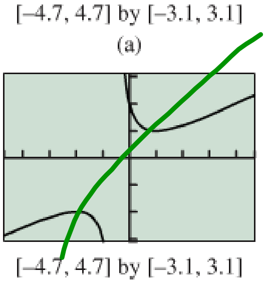
65. Which function matches the given equation?

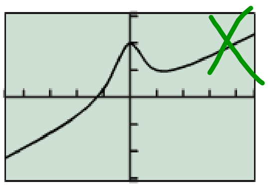
$$y = \frac{x+2}{2x^2+1}$$

D: $(-\infty, \infty)$


[–4.7, 4.7] by [–3.1, 3.1]
(a)


[–4.7, 4.7] by [–3.1, 3.1]
(b)


[–4.7, 4.7] by [–3.1, 3.1]
(c)


[–4.7, 4.7] by [–3.1, 3.1]
(d)

HW: Pg 95 #'s 48-72 by 3's, 81 and 84