Warm-Up

Find f(x) and g(x) so that y = g(f(x))

$y = \sqrt{x^2 - 5x}$

$f(x) = x^2 - 5x$

$g(x) = \sqrt{x}$

$y = e^{\sin x}$

$f(x) = \sin x$

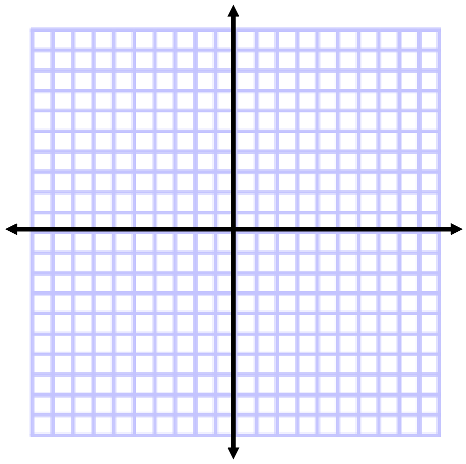
$g(x) = e^x$

Find two functions defined implicitly by:

$3x^2 - y^2 = 25$ Solve y

$y^2 = 3x^2 - 25$

$y = \pm\sqrt{3x^2 - 25}$



Sketch your two functions.

Section 1-5

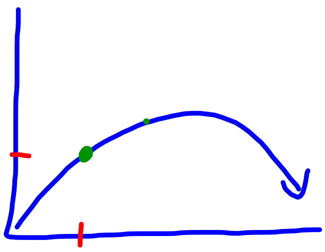
Parametrics Relations & Inverses

- Students will be able to define functions and relations parametrically
- Student will be able to find inverses of functions and relations

Relations Defined Parametrically

- Another natural way to define functions, or more generally, relations, is to define both elements of the ordered pair (x, y) in terms of another variable *t*, called a parameter.

time



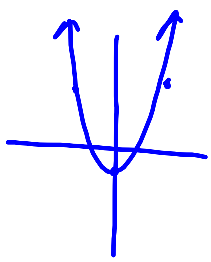
Defining a Function Parametrically

Consider the set of all ordered pairs (x, y) defined by the equations

$$x = t + 1 \qquad t = x - 1$$
$$y = t^2 + 2t$$

a. Find the points determined by t = -3, -2, -1, 0, 1, 2 & 3

t	x = t + 1	y = t^2 + 2t	(x, y)
-3	-2	3	(-2, 3)
-2	-1	0	(-1, 0)
-1	0	-1	(0, -1)
0	1	0	(1, 0)
1	2	3	(2, 3)
2	3	8	(3, 8)
3	4	15	(4, 15)



b. Find an algebraic relationship between x and y. Is y a function of x?

yes

$$y = (x-1)^2 + 2(x-1)$$
$$= x^2 - \cancel{2x} + 1 + \cancel{2x} - 2$$

$$y = x^2 - 1$$

c. Graph the relation in the (x, y) plane.

Inverse Function

If f is a one-to-one function with domain D and range R, then the **inverse function of f**, denoted f^{-1}, is the function with domain R and range D defined by

$$f^{-1}(b) = a \text{ if and only if } f(a) = b.$$

$f^{-1} \neq \frac{1}{f}$

$f(x)$ Function	$f^{-1}(x)$ Inverse
(x, y)	(y, x) switches (x,y)
Domain: D	Domain: R
Range: R	Range: D

How to Find an Inverse Function Algebraically

Given a formula for a function f, proceed as follows to find a formula for f^{-1}.

1. Determine that there is a function f^{-1} by checking that f is one-to-one.
   State any restrictions on the domain of f.
2. Switch x and y in the formula $y = f(x)$.
3. Solve for y to get the formula for $y = f^{-1}(x)$.
   State any restrictions on domain of f^{-1}.

Find the inverse of: f(x) = 3x - 2

$y = 3x - 2$

$x = 3y - 2$

$+2 \quad\quad +2$

$\frac{x+2}{3} = \frac{3y}{3}$

$\frac{x+2}{3} = y$

$f^{-1}(x) = \frac{x+2}{3}$

Example **Finding an Inverse Function Algebraically**

$y = f(x)$

Find an equation for $f^{-1}(x)$ if $f(x) = \dfrac{2x}{x-1}$.

$$y = \frac{2x}{x-1}$$

$$(y-1)\,x = \frac{2y}{\cancel{y-1}} \cdot \cancel{(y-1)}$$

$$yx - x = 2y$$

$$-yx \qquad\qquad -yx$$

$$-x = 2y - yx$$

$$\frac{-x}{2-x} = \frac{y(2-x)}{2-x}$$

$$\boxed{y = \frac{-x}{2-x}}$$

Finding an Inverse Function Algebraically

Find an equation for $f^{-1}(x)$ if $f(x) = \dfrac{x}{x+1}$

Find the inverses of the following functions & determine if the inverse is also a function.

1. $f{:}x \Rightarrow 4^x$

$f{:}x$

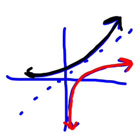
$f(x)$

y

*Switch x and y and solve for y

$y = 4^x$

$x = 4^y$



$\log_4 x = y$

$y = \dfrac{\log x}{\log 4}$

2. $f{:}x \Rightarrow \dfrac{2x + 3}{5}$

$y = \dfrac{2x}{5} + 3$

$x = \dfrac{2y}{5} + 3$

$x - 3 = \dfrac{2y}{5}$

$\dfrac{5(x-3)}{2} = \dfrac{2y}{2}$

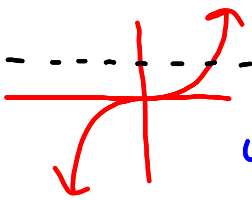
$\dfrac{5}{2}x - \dfrac{15}{2} = y$

$\dfrac{5(x-3)}{2} = y$

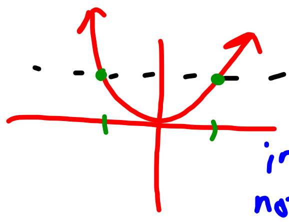
$\dfrac{5x - 15}{2} = y$

Inverse of a Function

- to determine if a function's inverse is also a function, perform the horizontal line test



inverse will be a function



inverse is not a function

- if a graph passes the horizontal line test, its inverse is a function
    - graphs of inverse functions reflect over the line y = x.
    - a function is a one-to-one function if it has an inverse.
      (1 - 1)

Horizontal Line Test

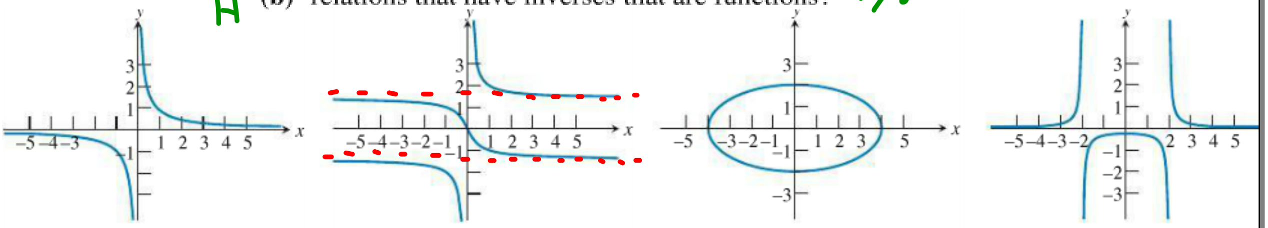
The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

EXAMPLE 3 Applying the Horizontal Line Test

Which of the graphs (1)–(4) in Figure 1.66 are graphs of

V **(a)** relations that are functions? 1,4

H **(b)** relations that have inverses that are functions? 1,2

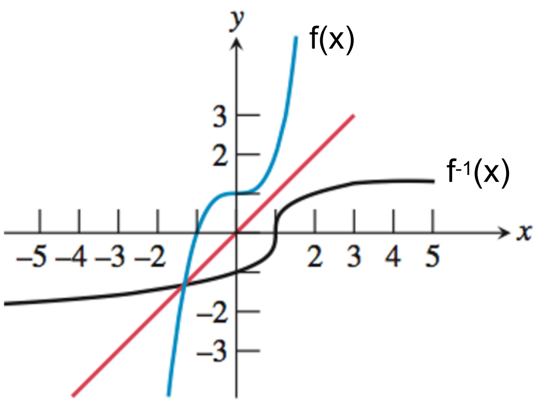


(1) (2) (3) (4)

1-1

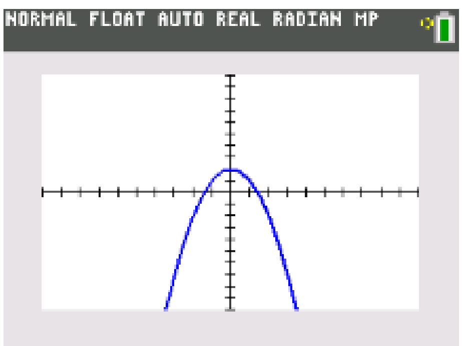
The Inverse Reflection Principle

The points (a, b) and (b, a) in the coordinate plane are symmetric with respect to the line $y = x$. The points (a, b) and (b, a) are **reflections** of each
2,7 7,2
other across the line $y = x$.



The reflection.

Find the graph of the inverse.



The Inverse Composition Rule

A function f is one-to-one with inverse function g if and only if
$f(g(x)) = x$ for every x in the domain of g, and
$g(f(x)) = x$ for every x in the domain of f.

Example Verifying Inverse Functions

Show algebraically the $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverse functions.

$f(g(x)) = (\sqrt[3]{x-2})^3 + 2$
$= x - 2 + 2$
$= x$

$g(f(x)) = \sqrt[3]{x^3+2-2}$
$= \sqrt[3]{x^3}$
$= x$

Inverse of a Function

- f(g(x)) = x for all the x in the domain of g
  g(f(x)) = x for all the x in the domain of f.
  - basically means if you plug x into the composite of a function and it's inverse, you'll get x as an answer

* if f and g are inverses then: $f \circ g(x) = x$ and $g \circ f(x) = x$

$f(g(x)) = x \qquad g(f(x)) = x$

$f(x) = x^3 - 1$

$g(x) = (x+1)^{\frac{1}{3}}$

$f(g(x)) = \left[(x+1)^{\frac{1}{3}}\right]^3 - 1$
$= x + 1 - 1$
$= x$

$g(f(x)) =$
$= (x^3 - 1 + 1)^{1/3}$
$= (x^3)^{1/3}$
$= x$

$f(x) = kx$

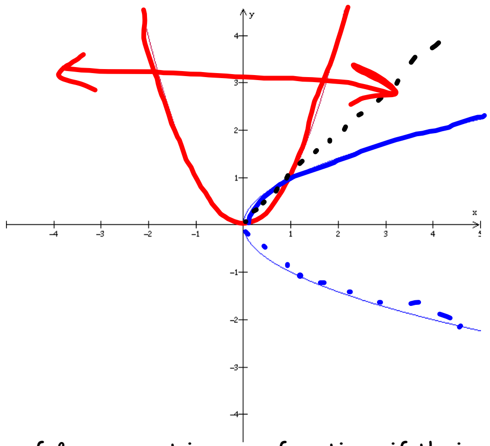
$g(x) = \dfrac{x}{k}$

$f(g(x)) = \cancel{k}\dfrac{x}{\cancel{k}}$
$= x$

$g(f(x)) = \dfrac{\cancel{k}x}{\cancel{k}}$
$= x$

Restricting Functions so Their Inverses Are Functions

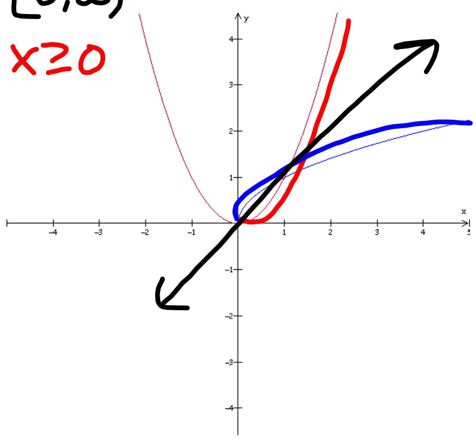
- some functions have no inverse over their entire domain, but they may have an inverse over a restricted domain.



f & g are not inverse functions if their domains are the set of all real numbers

$f: x \Rightarrow x^2$ $[0, \infty)$ $x \geq 0$

$g: x \Rightarrow \sqrt{x}$



If we limit the domain to just postive reals, then we're OK

Restricting domains is necessary if a function does not pass the horizontal line test.

1. Given the function f defined by $f(x) = \{(3, 4), (1, -2), (5, -1), (0, 2)\}$, find f^{-1} if it exists. If not, explain why not. $\{(4,3),(-2,1),(-1,5),(2,0)\}$
2. Name two points on the inverse of the function t, when $t(x) = x^5 + 3x^2 - 1$. $(-1, 0)$ $(9, 1)$

HW 1.5

#3 - 42 by 3's, #47

PG 126