Warm-Up

Find the formula for the inverse.

$$f(x)=\frac{x+5}{2x-3}$$

$$y=\frac{x+5}{2x-3}$$

$$f^{-1}(x)=$$

$$(2y-3)\,x=\frac{y+5}{\cancel{2y-3}}\cancel{(2y-3)}$$

$$2xy-3x=y+5$$
$$-5 \qquad -5$$
$$2xy-3x-5=y$$
$$-2xy \qquad -2xy$$
$$-3x-5=y-2xy$$
$$=y(1-2x)$$

$$y=\frac{-3x-5}{1-2x}$$

Multiple Choice Which ordered pair is in the *inverse* of the relation given by $x^2y + 5y = 9$?

(A) $(2, 1)$ **(B)** $(-2, 1)$ **(C)** $(-1, 2)$ **(D)** $(2, -1)$ **(E)** $(1, -2)$

y x

y x

$$(-2)^2(1)+5(1)$$
$$4+5$$
$$9$$

Section 1-6

Graphical Transformations

- Students will be able to algebraically and graphically represent translations, reflections, stretches and shrinks of functions.

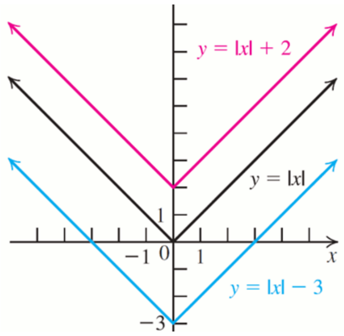
TRANSFORMATIONS

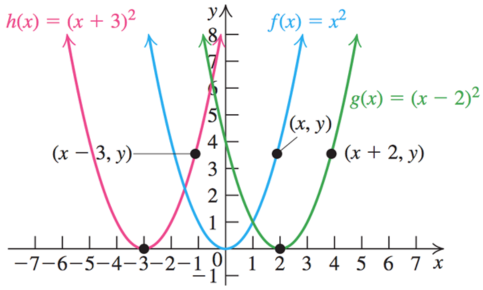
If a new function is formed by performing certain operations on a given function *f*, then the graph of the new function is called a **transformation** of the graph of *f*.

Reflections, and horizontal/vertical shifts are called **rigid transformations** because they do not change the shape of the graph.

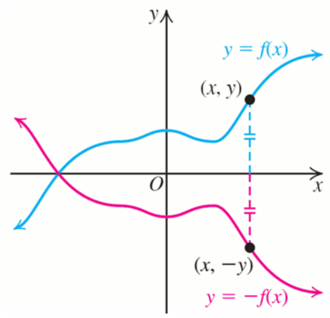
Vertical and Horizontal Shifts

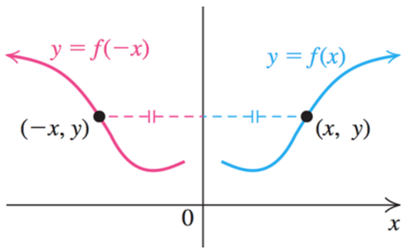
(Translations)





Reflections





Translations

Let c be a positive real number. Then the following transformations result in translations of the graph of $y = f(x)$.

Horizontal Translations

$y = f(x - c)$	a translation to the right by c units
$y = f(x + c)$	a translation to the left by c units

Vertical Translations

$y = f(x) + c$	a translation up by c units
$y = f(x) - c$	a translation down by c units

Vertical/Horizontal Translations

Describe how the graph of y = |x| can be transformed to the graph of the given equation.

$y = |x| - 4$

vertical translation of 4 units down

$y = |x + 2|$

horizontal translation of 2 units left

#3

#11

Describe how the graph of $y = x^2$ can be transformed to the graph of $y = (x - 3)^2$

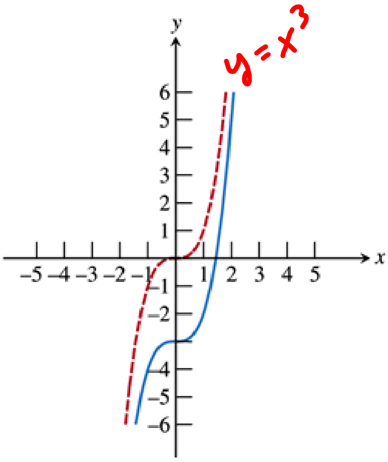
$y = (x-3)^2 - 5$

right 3

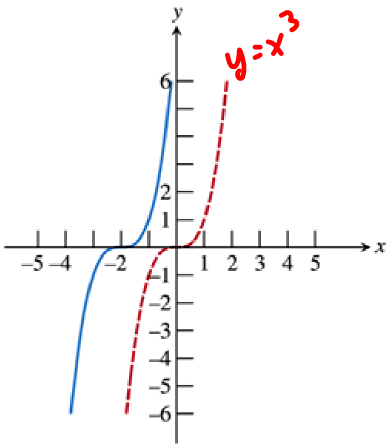
down 5

Finding Equations for Translations

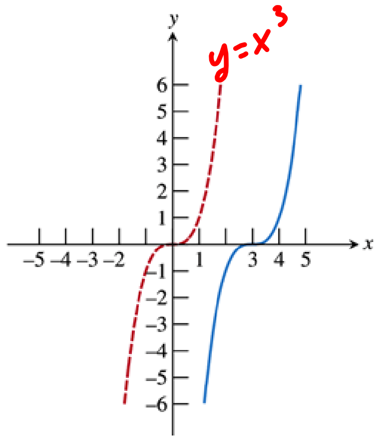
Write an equation for y_2 the translation of x^3 shown in the graphs...



$y_2 = x^3 - 3$

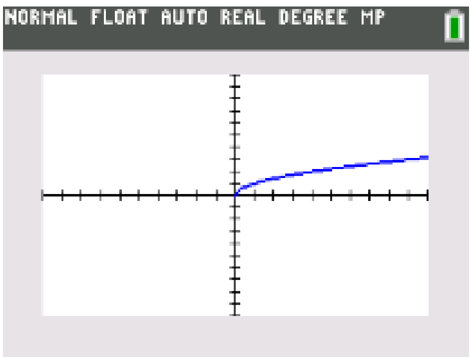


$y_2 = (x+2)^3$

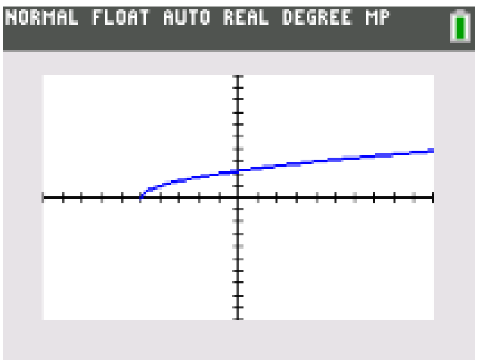


$y_2 = (x-3)^3$

Write the formula for the function graphed that was obtained by performing a translation on $y = \sqrt{x}$



$y = \sqrt{x}$



$y = \sqrt{x+5}$

Reflections Across Axes

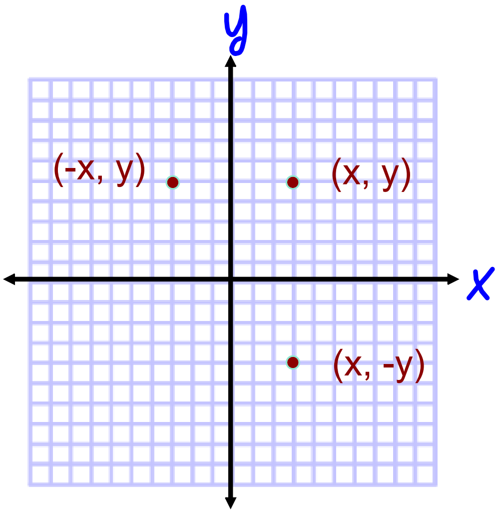
The following transformations result in reflections of the graph of $y = f(x)$:

Across the x-axis

$y = -f(x)$

Across the y-axis

$y = f(-x)$



Finding Equations for Reflections

Find an equation for the reflection of $f(x) = \dfrac{5x - 9}{x^2 + 3}$ across each axis.

x-axis $y = -f(x)$ $\quad -1\left(\dfrac{5x-9}{x^2+3}\right) \quad \dfrac{-5x+9}{x^2+3}$

y-axis $y = f(-x) = \dfrac{5(-x)-9}{(-x)^2+3} \quad \dfrac{-5x-9}{x^2+3}$

Find the equation of the reflection of f across the x-axis and y-axis

$$f(x) = x^3 - 5x^2 - 3x + 2$$

x-axis $= -(x^3 - 5x^2 - 3x + 2)$
$= -x^3 + 5x^2 + 3x - 2$

y-axis $= (-x)^3 - 5(-x)^2 - 3(-x) + 2$
$= -x^3 - 5x^2 + 3x + 2$

Stretches & Shrinks

Let c be a positive real number. Then the following transformations result in stretches or shrinks of the graph of $y = f(x)$:

Horizontal Stretches or Shrinks

$\frac{1}{5}x = \frac{x}{5}$

$$y = f\left(\frac{x}{c}\right) \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$

Vertical Stretches or Shrinks

$$y = c \cdot f(x) \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$

Vertical/horizontal stretching and compressing (shrinking) on a graph distort the shape, so they are called **nonrigid transformations**.

Vertical Stretch/Compress $y = c \cdot f(x)$

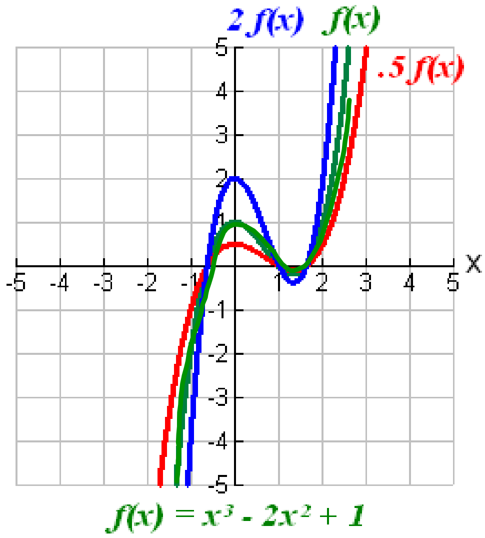
$c > 1$ Stretch/

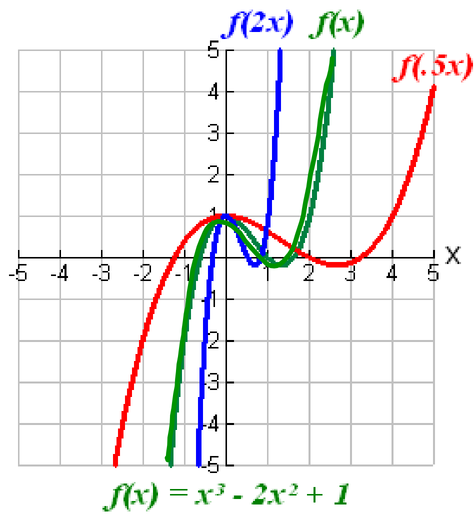
$c < 1$ Compress

Horizontal Stretch/Compress $y = f\left(\frac{x}{c}\right)$

$c > 1$ Stretch/

$c < 1$ Compress





For the problems below, describe the transformations that produce the graph of g from the graph of f. Note - if there is a number in front of the x term, factor it out.

1. $f(x) = \sqrt{x}$

   $g(x) = -\sqrt{x+4} - 3$

   reflect over x
   horizontal translation 4 left
   vertical translation 3 down

2. $f(x) = x^3$

   $g(x) = \frac{1}{4}\boxed{(-x+1)}^3 + 2$

   $\frac{1}{4}\boxed{(-(x-1))^3} + 2$

   vertical shrink $\frac{1}{4}$
   reflect y-axis
   right 1 unit
   up 2 units

3. $f(x) = |x|$

   $g(x) = 5|3x-6|$

   $= 5|3(x-2)|$

   vertical

   $\frac{x}{3}$

   5 vertical stretch
   3 horiz shrink
   2 units right

Write an equation for a function whose graph fits the given description.

The graph of f(x) = $\sqrt{x}$ is shifted two units down, reflected in the x axis, and compressed vertically by a factor of 1/2.

$y = \frac{1}{2}\sqrt{x}$ $\quad y = -\frac{1}{2}\sqrt{x}$ $\quad \boxed{y = -\frac{1}{2}\sqrt{x} - 2}$

The graph of g(x) = x^3 is shifted four units left, stretched vertically by a factor of 3, reflected in the y-axis and shifted two units up.

$y = 3(-x-4)^3 + 2 \qquad 3(-(x+4))^3 + 2$

The graph of h(x) = $|x|$ is shifted six units right, stretched horizontally by a factor of 1/3, reflected in the x-axis, and shifted three units down.

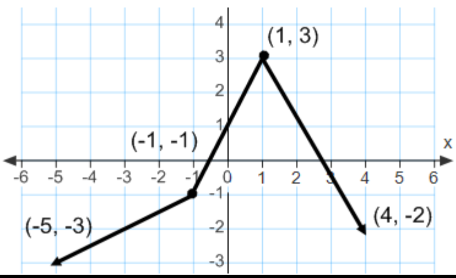
$y = |x-6|$ $\qquad y = -\left|\frac{1}{3}(x-6)\right|$

$y = \left|\frac{1}{3}(x-6)\right|$ $\qquad \boxed{y = -\left|\frac{1}{3}(x-6)\right| - 3}$

When you combine transformations, it is best to perform them in the following order:

1. Horizontal Shifts, 2. Stretch or Compress, 3. Reflections, 4. Vertical Shifts

Given the graph f(x) to the right, graph the following function:

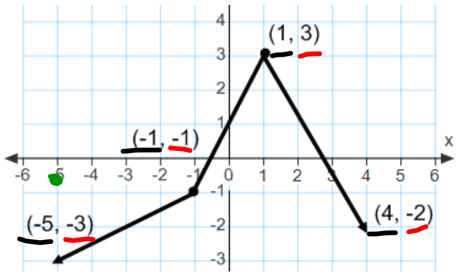


$g(x) = \frac{1}{2}f(x) + 1$

x	f(x)
-5	-3
-1	-1
1	3
4	-2

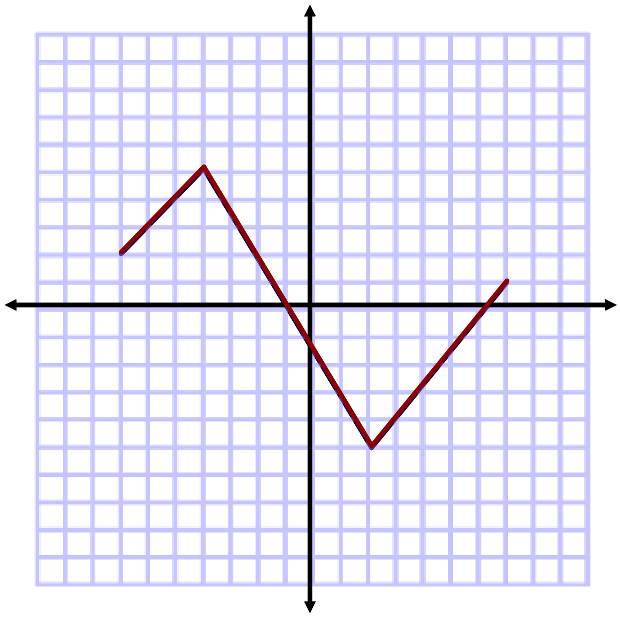
g(x)

$\frac{1}{2}(-3) + 1 = -\frac{1}{2}$



Transforming a Graph Geometrically

Decribe how to transform the graph of y = f(x) shown to the graph of y = - f(x - 2) + 4



Summary of Transformations:

(Below, the brackets represent the basic function)

$$y = - a [\overbrace{- b (x + c}^{\text{opposites}})] + d$$

reflect over x-axis

vertical stretch/shrink (stretch if a > 1) (shrink if a < 1)

reflect over y-axis

horizontal stretch/shrink (stretch if b < 1) (shrink if b > 1)

horizontal shift
(right if c < 0) -
(left if c > 0) +

vertical shift
(up if d > 0) +
(down if d < 0) -

Section 1.6 Homework:

3 - 30, 39 - 54 by 3's

59 - 64 all

#15 stretch/shrink

Summary of Transformations:

(Below, the brackets represent the basic function)

$$y = -a\,[-b\,(x + c)] + d$$

reflect over x-axis

vertical stretch/shrink (stretch if a > 1) (shrink if a < 1)

reflect over y-axis

horizontal stretch/shrink (stretch if b < 1) (shrink if b > 1)

horizontal shift (right if c < 0) (left if c > 0)

vertical shift (up if d > 0) (down if d < 0)