Summary of Transformations:

(Below, the brackets represent the basic function)

$$y = -a[-b(x + c)] + d$$

- reflect over x-axis
- reflect over y-axis
- horizontal shift (right if c < 0) (left if c > 0)
- vertical shift (up if d > 0) (down if d < 0)
- vertical stretch/shrink (stretch if a > 1) (shrink if a < 1)
- horizontal stretch/shrink (stretch if b < 1) (shrink if b > 1)

Warm-Up

describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

$y = -2|x + 4| + 1$

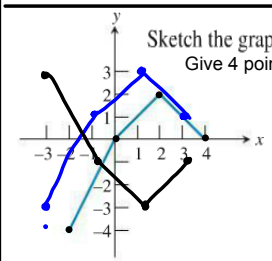
reflect x-axis
vertical stretch
horizontal shift 4 left
vertical shift 1 up

Write an equation that has the following:

$y = |x|$: a shift left 2 units, then a horizontal shrink by a factor of 1/2, and finally a shift down 4 units.

$y = |x+2|$
$y = |2x+2|$
$y = |2x+2| - 4$

Sketch the graph of $y = -f(x + 1) + 1$.
Give 4 points on your final graph.

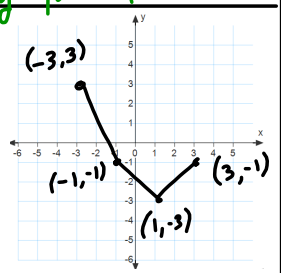


left 1 up 1

reflect x

$y = -f(x+1)+1$

x	f(x)
-2	-4
0	0
2	2
4	0



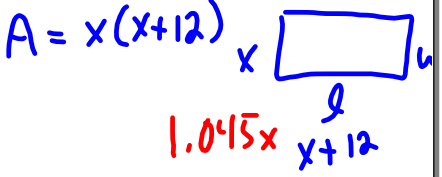
Section 1-7

Modeling with Functions

- Identify appropriate basic functions with which to model real-world problems.
- Produce specific functions to model data, formulas, graphs, and verbal descriptions.

Write a mathematical expression.

1. Five more than three times a number x — $3x + 5$
2. A number x increased by 5 and then tripled — $3(x+5)$
3. Seventeen percent of a number x — $0.17x$
4. Four more than 5% of a number x — $4 + .05x$
5. **Area of a Rectangle** The area of a rectangle whose length is 12 more than its width x — $A = x(x+12)$



6. **Salary Increase** A salary after a 4.5% increase, if the original salary is x dollars — $1.045x$
7. **Income Loss** Income after a 3% drop in the current income of x dollars — $0.97x$

Forming Functions from Formulas

Write the area A of a circle as a function of its...

a. radius; *r*

$A = \pi r^2$

b. diameter; *d*

$d = 2r$ $A = \pi\left(\frac{d}{2}\right)^2$

$r = \frac{d}{2}$ $= \pi\frac{d^2}{4}$ or $\frac{\pi}{4}d^2$

c. circumference; *C*

$A = \pi\left(\frac{C}{2\pi}\right)^2 = \pi\frac{C^2}{4\pi^2} = \frac{C^2}{4\pi}$

$C = 2\pi r$

$r = \frac{C}{2\pi}$

#11

1. **Area of a Triangle** $A = \frac{1}{2}bh$
2. **Area of a Trapezoid** $A = \frac{1}{2}(b_1 + b_2)h$
3. **Volume of a Right Circular Cylinder** $V = \pi r^2 h$
4. **Volume of a Right Circular Cone** $V = \frac{1}{3}\pi r^2 h$
5. **Volume of a Sphere** $V = \frac{4}{3}\pi r^3$
6. **Surface Area of a Sphere** $A = 4\pi r^2$
7. **Surface Area of a Right Circular Cylinder** $A = 2\pi rh + 2\pi r^2$
8. **Simple Interest** $I = Prt$
9. **Compound Interest** $A = P\left(1 + \frac{r}{n}\right)^{nt}$
10. **Free Fall from Height *H*** $s = H - \frac{1}{2}gt^2$

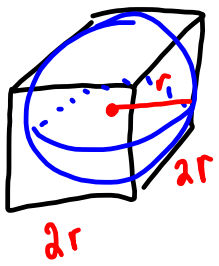
A sphere is contained in a cube, tangent to all six faces. Find the surface area of the cube as a function of the radius of the sphere.

$SA_{cube} = s^2 \cdot 6$

$= (2r)^2 \cdot 6$

$= 4r^2 \cdot 6$

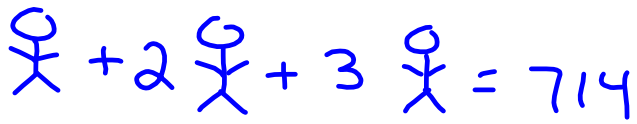
$=$



$SA_{cube} = 6(2r)(2r)$

$= 24r^2$

When a number is added to its double and its triple, the sum is 714. Find the three numbers.



$x + 2x + 3x = 714$

$\frac{6x}{6} = \frac{714}{6}$

$x = 119$

x	2x	3x
119	238	357

Job Offers Ruth is weighing two job offers from the sales departments of two competing companies. One offers a base salary of \$25,000 plus 5% of gross sales; the other offers a base salary of \$20,000 plus 7% of gross sales. What would Ruth's gross sales total need to be to make the second job offer more attractive than the first?

$25000 + 0.05x < 20000 + .07x$

$-20000 \quad -20000$

$5000 + .05x < .07x$

$-.05x \quad -.05x$

$5000 < .02x$

$\frac{5000}{.02} < \frac{.02x}{.02}$

\$250,000 < x

Mixing Solutions How much 10% solution and how much 45% solution should be mixed together to make 100 gal of 25% solution?

$.1x + .45(100 - x) = 100(.25)$

$.1x + 45 - .45x = 25$

$-.35x + 45 = 25$

$-45 \quad -45$

$-.35x = -20$

$x = 57.14$ gal of 10% soln

$100 - 57.14$ 42.86 gal of 45% soln

#32 $.2x + .35(25 - x) = .26(25)$

$.2x + 8.75 - .35x = 6.5$

$8.75 - .15x = 6.5$

$-8.75 \quad -8.75$

$-.15x = -2.25$

$x = 15$ L of 20% soln

10 L of 35% soln

Letting Units Work for You

How many rotations does a 15 in. (radius) tire make per second on a sport utility vehicle traveling 70 mph?

$C = 2\pi r$
$= 2\pi(15)$
$= 30\pi$ in

$\frac{1 \text{ rot}}{30\pi \text{ in}}$

$$\frac{70 \text{ miles}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ rot}}{30\pi \text{ in}}$$

$$\frac{70(5280)(12)}{(60)(60)(30\pi)} = 13 \frac{\text{rot}}{\text{sec}}$$

Curve-fitting technology

The table shows that the number of patent applications in the US increased from 1993 to 2003. Find both a linear and quadratic regression model for this data. Which appears to be the better model of the data?

U.S. Patent Applications

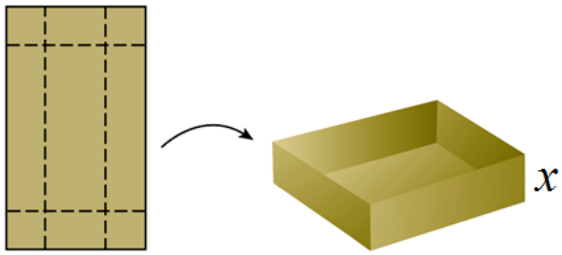
Year	Applications (thousands)
1993	189.4
1994	206.9
1995	226.6
1996	211.6
1997	233.0
1998	261.5
1999	289.5
2000	315.8
2001	346.6
2002	357.5
2003	367.0

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States*, 2003 (Washington, D.C., 2003).

#47

A Maximum Value Problem

A square of side x inches is cut out of each corner of an 8 in. by 15 in. piece of cardboard and the sides are folded up to form an open topped box.



a. Write the volume V of the box as a function of x.

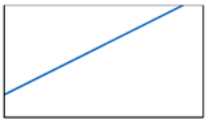
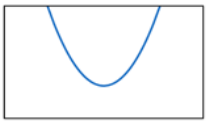
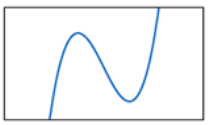
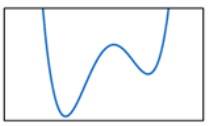
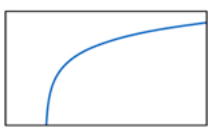
b. Find the domain of V as a function of x.

c. Graph V as a function of x over the domain found in part b and find the maximum volume the box can hold

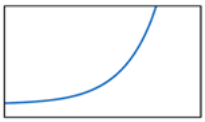
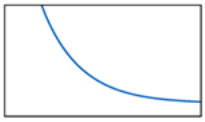
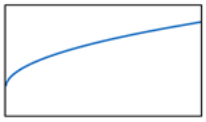
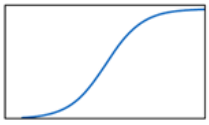
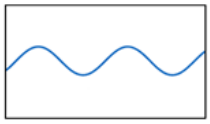
d. How big should the cut-out square be in order to produce the box of maximum volume?

#33

Functions

Regression Type	Equation	Graph	Applications
Linear (Chaper 2)	$y = ax + b$		Fixed cost plus variable cost, linear growth, free-fall velocity, simple interest, linear depreciation, many others
Quadratic (Chapter 2)	$y = ax^2 + bx + c$ (requires at least 3 points)		Position during free fall, projectile motion, parabolic reflectors, area as a function of linear dimension, quadratic growth, etc.
Cubic (Chapter 2)	$y = ax^3 + bx^2 + cx + d$ (requires at least 4 points)		Volume as a function of linear dimension, cubic growth, miscellaneous applications where quadratic regression does not give a good fit
Quartic (Chapter 2)	$y = ax^4 + bx^3 + cx^2 + dx + e$ (requires at least 5 points)		Quartic growth, miscellaneous applications where quadratic and cubic regression do not give a good fit
Natural logarithmic (ln) (Chapter 3)	$y = a + b \ln x$ (requires $x > 0$)		Logarithmic growth, decibels (sound), Richter scale (earthquakes), inverse exponential models

Functions cont.

Regression Type	Equation	Graph	Applications
Exponential ($b > 1$) (Chapter 3)	$y = a \cdot b^x$		Exponential growth, compound interest, population models
Exponential ($0 < b < 1$) (Chapter 3)	$y = a \cdot b^x$		Exponential decay, depreciation, temperature loss of a cooling body, etc.
Power (requires $x, y > 0$) (Chapter 2)	$y = a \cdot x^b$		Inverse-square laws, Kepler's third law
Logistic (Chapter 3)	$y = \frac{c}{1 + a \cdot e^{-bx}}$		Logistic growth: spread of a rumor, population models
Sinusoidal (Chapter 4)	$y = a \sin(bx + c) + d$		Periodic behavior: harmonic motion, waves, circular motion, etc.

Section 1.7 Homework:

Pg 148. #9, 12, 15, 18, 21, 27, 32, 36, 39, 42-46