Chapter 3
Laws of Algebra

- Laws of Exponents
- The Distributive Law
- The product (a + b)(c + d)
- Difference of two squares
- Perfect squares expressions
- Further expansion

$a^m \times a^n = a^{m+n}$	To **multiply** numbers with the **same base**, keep the base and **add** the exponents.
$\frac{a^m}{a^n} = a^{m-n}, \ a \neq 0$	To **divide** numbers with the **same base**, keep the base and **subtract** the exponents.
$(a^m)^n = a^{m \times n}$	When **raising** a **power** to a **power**, keep the base and **multiply** the exponents.
$(ab)^n = a^n b^n$	The power of a product is the product of the powers.
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \ b \neq 0$	The power of a quotient is the quotient of the powers.
$a^0 = 1, \ a \neq 0$	Any non-zero number raised to the exponent zero is 1.
$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ and in particular $a^{-1} = \frac{1}{a}, \ a \neq 0.$	

$q^{-3} = \frac{1}{q^3}$

Simplify: $p^3 \times p^8 = p^{11}$ $3^5 \times 3^4 = 3^9$

Simplify: $\frac{x^7}{x^3} = x^{7-3} = x^4$

$\frac{b^2}{b^9} = b^{2-9} = b^{-7} = \frac{1}{b^7}$

Simplify: $(c^3)^5$ c^{15}

$(3d^4)^3$ $3^3 d^{12}$ $27d^{12}$

Simplify using $a^m \times a^n = a^{m+n}$

a $k^4 \times k^2$ k^6

b $5^2 \times 5^6$ 5^8

c $d^3 \times d^7$ d^{10}

d $11^4 \times 11^a$ 11^{4+a}

Simplify using $\frac{a^m}{a^n} = a^{m-n}$

e $\frac{k^{12}}{k^a}$ k^{12-a}

f $\frac{y^6}{y}$ y^5

g $\frac{t^m}{t^4}$ t^{m-4}

h $\frac{x^{3a}}{x^2}$ x^{3a-2}

Simplify using $(a^m)^n = a^{mxn}$

a $(5^3)^2$ 5^6

b $(c^4)^3$ c^{12}

c $(3^8)^4$ 3^{32}

d $(v^5)^5$ v^{25}

Simplify

a $b^5 \times b^7$ b^{12}

b $\frac{t^9}{t^2}$ t^7

c $(p^6)^3$ p^{18}

d $\frac{7^6}{7^n}$ 7^{6-n}

e $(x^{2s})^3$ x^{6s}

f $d^k \div d^3$ $\frac{d^k}{d^3}$ d^{k-3}

g $3^2 \times 3^7 \times 3^4$ 3^{13}

h $(j^4)^{3x}$ j^{12x}

i $11^6 \times 11^1$ 11^7

j $\frac{z^7}{z^{4t}}$ z^{7-4t}

k $(13^c)^{5d}$ 13^{5cd}

l $w^{7p} \div w^1$ w^{7p-1}

Write as powers of 2:

16 $= 2^4$

$\frac{1}{16} = \frac{1}{2^4}$

4 x 2^n

$2^2 \times 2^n$

2^{2+n}

$\frac{2^m}{8} = \frac{2^m}{2^3}$

2^{m-3}

Write as powers of 2:

a $2^1 \times 2^a$ 2^{1+a}

b 4×2^b $2^2 \cdot 2^b$ 2^{2+b}

c 8×2^t $2^3 \cdot 2^t = 2^{3+t}$

f $\frac{2^c}{4} = \frac{2^c}{2^2}$ $= 2^{c-2}$

g $\frac{2^m}{2^{-m}}$ 2^{m--m} 2^{m+m} 2^{2m}

h $\frac{4}{2^{1-n}} = \frac{2^2}{2^{1-n}} = 2^{2-(1-n)}$ 2^{2-1+n} 2^{1+n}

Write as powers of 3:

a 9×3^p $3^2 \cdot 3^p$ 3^{2+p}

b 27^a $(3^3)^a$ 3^{3a}

c 3×9^n $= 3 \cdot (3^2)^n$ $3^1 \cdot 3^{2n} = 3^{1+2n}$

f $\frac{3^y}{3}$ 3^{y-1}

g $\frac{3}{3^y}$ 3^{1-y}

h $\frac{9}{27^t}$ $\frac{3^2}{(3^3)^t} = \frac{3^2}{3^{3t}} = 3^{2-3t}$

Write without brackets

$(4x)^3$ $4^3 x^3$ $64x^3$

$\frac{x^3y^4}{z^2}$ $\frac{x^6y^8}{z^4}$

a $(2a)^2$ $4a^2$

b $(3n)^2$ $9n^2$

c $(5m)^3$ $5^3m^3 = 125m^3$

e $\left(\frac{a}{2}\right)^3 = \frac{a^3}{2^3} = \frac{a^3}{8}$

f $\left(\frac{3}{m}\right)^2 = \frac{3^2}{m^2} = \frac{9}{m^2}$

g $\left(\frac{p}{q}\right)^4 = \frac{p^4}{q^4}$

Write without negative exponents

x^{-4} $\frac{1}{x^4}$

5^{-2} $\frac{1}{5^2} = \frac{1}{25}$

$\left(\frac{x}{2}\right)^{-3} = \frac{x^{-3}}{2^{-3}} = \frac{2^3}{x^3}$ $= \frac{8}{x^3}$

Simplify

$3x^{-2}y^{5}\cdot 4^{-2}x^{6}y^{-8}$ $3\cdot 4^{-2}\cdot x^{-2+6}\cdot y^{5+-8}$

$3\cdot 4^{-2}x^{4}y^{-3}$

$\frac{3y^{5}x^{6}}{x^{2}4^{2}y^{8}}$ $\frac{3x^{4}}{16y^{3}}$

$\frac{3x^{6}y^{5}}{16x^{2}y^{8}} = \frac{3x^{6-2}y^{5-8}}{16} = \frac{3x^{4}y^{-3}}{16} = \frac{3x^{4}}{16y^{3}}$

$\frac{m^{-3}n^{5}p^{2}}{m^{3}n^{8}p^{-4}} = \frac{n^{5}p^{6}}{m^{6}}$

$\left(\frac{4x^{2}y^{-3}}{3x^{5}y^{-6}}\right)^{2}$ $\frac{4^{2}x^{4}y^{-6}}{3^{2}x^{10}y^{-12}} = \frac{16y^{6}}{9x^{6}}$

$\frac{y^{12}}{y^{6}}$

$\left(\frac{2x^{5}y^{4}}{5x^{-3}y^{-2}}\right)^{-3} = \left(\frac{5x^{-3}y^{-2}}{2x^{5}y^{4}}\right)^{3} = \frac{5^{3}x^{-9}y^{-6}}{2^{3}x^{15}y^{12}}$

$\frac{2^{-3}x^{-15}y^{-12}}{5^{-3}x^{9}y^{6}}$ $= \frac{125}{8x^{24}y^{18}}$