H   Error and Percentage Error

An **approximation** is a value given to a number which is close to, but not equal to, its true value.

Approximations often occur when we round off results obtained by measurement.
For example, $36.428\,97$ is approximately 36.4.

An **estimation** is a value which has been found by judgement or prediction instead of carrying out a more accurate measurement.

For example, we can estimate 38.7×5.1 to be $40 \times 5 = 200$ whereas its true value is 197.37. A good approximation of this true value would be 197.

In order to make reasonable estimations we often appeal to our previous experience.

ERROR

Whenever we measure a quantity there is almost always a difference between our measurement and the actual value. We call this difference the **error**.

If the actual or exact value is V_E and the approximate value is V_A then the

$$\text{Error} = 200 - 197.37$$
$$\text{Error} = 2.63$$

$$\text{error} = V_A - V_E$$ ↗ Appr.   ↘ exact

Error is often expressed as a percentage of the exact value, and in this case we use the *size* of the error, ignoring its sign. We therefore use the **modulus** of the error.

$$\boxed{\text{Percentage error } E = \frac{|V_A - V_E|}{V_E} \times 100\%}$$

$$\frac{|2.63|}{197.37} \cdot 100$$
$$(0.1332)100$$
$$1.3\%$$

Example 20   ◀ Self Tutor

You estimate a fence's length to be 70 m whereas its true length is 78.3 m. Find, correct to one decimal place:

a the error   **b** the percentage error.

a $\text{error} = V_A - V_E$
$\quad = 70 - 78.3$
$\quad = -8.3$ m

b percentage error
$\quad = \frac{|V_A - V_E|}{V_E} \times 100\%$
$\quad = \frac{|-8.3|}{78.3} \times 100\%$
$\quad \approx 10.6\%$

$.100\%$

Example 21   🔊 **Self Tutor**

Alan wants to lay carpet on his 4.2 m by 5.1 m lounge room floor. He estimates the area of the lounge room by rounding each measurement to the nearest metre.

a Find Alan's estimate of the lounge room area.

b The carpet costs $39 per square metre. Find the cost of the carpet using Alan's estimate of the area.

c Find the actual area of Alan's lounge room.

d Find the percentage error in Alan's estimation.

e Will Alan have enough carpet to cover his lounge room? How should he have rounded the measurements?

a Area $\approx 4 \text{ m} \times 5 \text{ m}$ **b** Cost $= 20 \times \$39$ **c** Actual area $= 4.2 \times 5.1$
$\approx 20 \text{ m}^2$ $= \$780$ $= 21.42 \text{ m}^2$
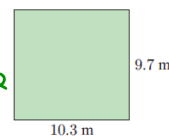
d Percentage error $= \dfrac{|V_A - V_E|}{V_E} \times 100\%$

$= \dfrac{|20 - 21.42|}{21.42} \times 100\%$

$\approx 6.63\%$

e Alan only has 20 m² of carpet, so he will not have enough to cover his lounge room.
He should have rounded the measurements *up* to make sure he had enough carpet.

EXERCISE 2H

1 Find   **i** the error   **ii** the percentage error   in rounding:
  a the yearly profit of €1 367 540 made by a company to €1.37 million
  b a population of 31 467 people to 31 000 people
  c a retail sales figure of $458 110 to $460 000
  d the number of new cars sold by a company in a year from 2811 to 3000.

2 Find   **i** the error   **ii** the percentage error   if you estimate:
  a the mass of a brick to be 5 kg when its actual mass is 6.238 kg
  b the perimeter of a property to be 100 m when its actual length is 97.6 m
  c the capacity of a container to be 20 L when its actual capacity is 23.8 L
  d the time to write a computer program to be 50 hours when it actually takes 72 hours.

3 Jon's lounge room is a 10.3 m by 9.7 m rectangle.
  a Estimate the floor area by rounding each length to the nearest metre. $10\,m \times 10\,m = 100\,m^2$
  b Find the actual area of the floor. $10.3\,m \times 9.7\,m = 99.91\,m^2$
  c What is the error in your calculation in **a**? $E = |100 - 99.91|$ $= .09$
  d What percentage error was made?

$\dfrac{1.091 \cdot .100}{99.91} =$

$9.008107297\,E\text{-}4 = {}^{\cdot 100}$

$9.008107297 \cdot 10^{-4} \quad {}^{\cdot pc}$

$.0009008107297 \cdot 100$

$.09\%$

9.7 m

10.3 m

4 The cost of freight for a parcel is dependent on its volume. Justine lists the dimensions of a parcel as 24 cm by 15 cm by 9 cm on the consignment note.
The actual dimensions are 23.9 cm × 14.8 cm × 9.2 cm.

   a Calculate the actual volume of the parcel.

   b Calculate the volume given on the consignment note.

   c Find the rounding error in the calculation.

   d What percentage error was made?

5 A hotel wants to cover an 8.2 m by 9.4 m rectangular courtyard with synthetic grass. The manager estimates the area by rounding each measurement to the nearest metre.

   a Find the manager's estimate of the area.

   b The synthetic grass costs $85 per square metre. Find the cost of the grass.

   c Find the actual area of the rectangle.

   d Calculate the percentage error in the manager's estimate.

   e Will the hotel have enough grass to cover the courtyard?

   f Find the cost of the grass if the manager had rounded each measurement *up* to the next metre.

6 The Italian flag is split into three equal sections.

   a Find, rounded to 1 decimal place, the length AB.

   b Use your rounded value in **a** to estimate the area of the green section.

   c Find the actual area of the green section.

   d Find the percentage error in the estimate in **b**.