1) $\int \frac{1}{3} e^{-x} \sin 2x \, dx$

$\frac{1}{3}\left[\int e^{-x} \sin(2x)\, dx\right]$

$u = e^{-x}$ $dv = \sin(2x)\, dx$

$du = -e^{-x} dx$ $v = -\frac{1}{2}\cos(2x)$

$uv - \int v\, du$

$-\frac{1}{2} e^{-x} \cos(2x) - \frac{1}{2}\int e^{-x} \cos(2x)\, dx$

$u = e^{-x}$ $dv = \cos 2x \, dx$

$du = -e^{-x} dx$ $v = \frac{1}{2}\sin 2x$

$\frac{1}{3}\int e^{-x}\sin 2x = \frac{1}{3}\left[-\frac{1}{2} e^{-x}\cos(2x) - \frac{1}{2}\left(\frac{1}{2} e^{-x} \sin 2x + \frac{1}{2}\int e^{-x} \sin 2x \, dx\right]\right.$

$\frac{1}{3}\int e^{-x} \sin 2x = -\frac{1}{6} e^{-x}\cos(2x) - \frac{1}{12} e^{-x} \sin 2x - \frac{1}{12}\int e^{-x} \sin 2x \, dx$

$\frac{5}{12}\int e^{-x} \sin 2x = -\frac{1}{6} e^{-x} \cos 2x - \frac{1}{12} e^{-x} \sin 2x + C$

$= -\frac{2}{5} e^{-x} \cos(2x) - \frac{1}{5} e^{-x} \sin(2x) + C$

$\int \ln x \, dx$

$u = \ln x$ $dv = 1\, dx$

$du = \frac{1}{x} dx$ $v = x$

$x \ln x - \int 1 \, dx$

$\int 6x \sin(3x^2)\, dx$

$u = 3x^2$ $du = 6x\, dx$

$\int \sin u \, du$

$-\cos u + C$

$\frac{d}{dx}(5^x) = 5^x \ln 5$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\log_3 x) = \frac{1}{x} \cdot \frac{1}{\ln 3}$

$\int \frac{1}{x \ln 7} dx = \log_7 x + C$

$\frac{1}{\ln 7}\int \frac{1}{x} dx$

$\frac{1}{\ln 7} \cdot \ln x = \frac{\ln_e x}{\ln_e 7} = \log_7 x$

$\frac{\log_{102} x}{\log_{102} 7}$