2.8 Rewrite Equations and Formulas

Before You wrote functions and used formulas.

Now You will rewrite equations and formulas.

Why? So you can solve a problem about bowling, as in Ex. 33.

Key Vocabulary
- **literal equation**
- **formula**

The equations $2x + 5 = 11$ and $6x + 3 = 15$ have the general form $ax + b = c$. The equation $ax + b = c$ is called a **literal equation** because the coefficients and constants have been replaced by letters. When you solve a literal equation, you can use the result to solve any equation that has the same form as the literal equation.

LESSON 2.8

Practice A

For use with the lesson "Rewrite Equations and Formulas"

Determine whether the equation is in function form.

1. $2x + y = 8$ **2.** $x = 3y - 4$ **3.** $y = 1 - 8x$

$-2x + 2x + y = 8 - 2x$

$y = -2x + 8$

YES

$+4 \quad x = 3y - 4 + 4$

$\frac{1}{3}(x + 4) = 3y \cdot \frac{1}{3}$

$\frac{1}{3}(x + 4) = y$

$y = \frac{1}{3}(x + 4)$

$y = \frac{1}{3}x + \frac{4}{3}$

Write the equation in function form.

4. $y + 10x = 3$

5. $y - 13 = 4x$

6. $8x + y - 4 = 0$

$13 + y - 13 = 4x + 13$

$y = 4x + 13$

7. $4x + 2y = 14$ **8.** $3y - 9x = 27$ **9.** $16 + 2y = 18x$

$4x + 2y = 14 + {}^{-}4x$

$\frac{1}{2} \cdot 2y = (-4x + 14)\frac{1}{2}$

$y = -2x + 7$

$16 + 2y = 18x$

$\frac{1}{2}\, 2y = (18x + {}^{-}16)\frac{1}{2}$

$y = 9x + {}^{-}8$

10. $15x - 5y = 20$ **11.** $2x - 3y = 6$ **12.** $24 - 4y = 8x$

$$-2x + 2x - 3y = 6 - 2x$$

$$-\frac{1}{3} \cdot -3y = \left(-\frac{2}{1}x + 6\right)\left(-\frac{1}{3}\right)$$

$$y = \frac{2}{3}x + -2$$

13. $5x + 2y = 16$

14. $-7x - 3y = 18$

15. $4y - 4x + 4 = 0$

Solve the literal equation.

16. Solve $P = R - C$ for C.

17. Solve $F = ma$ for m.

$$\frac{1}{A} \cdot \frac{F}{1} = mA \cdot \frac{1}{A}$$

$$\frac{F}{A} = m$$

m

18. Solve $I = \frac{E}{R}$ for R.

19. Solve $ax - by = c$ for x.

$$By + Ax - By = C + By$$

$$\frac{1}{A} Ax = (By + C) \cdot \frac{1}{A}$$

$$x = \frac{By + C}{A}$$

$$x = \frac{By}{A} + \frac{C}{A}$$

Solve the formula for the indicated variable.

20. Circumference of a circle: $C = 2\pi r$. Solve for r.

$$\frac{1}{2\pi} \cdot \frac{C}{1} = 2\pi r \cdot \frac{1}{2\pi}$$

$$\frac{C}{2\pi} = r$$

$$r = \frac{C}{2\pi}$$

21. Volume of a pyramid: $V = \frac{Bh}{3}$. Solve for B.

$$\frac{3}{h} \cdot V = \frac{Bh}{3} \cdot \frac{3}{h}$$

$$\frac{3V}{h} = B$$

$$B = \frac{3V}{h}$$

22. Perimeter of a rectangle: $P = 2\ell + 2w$. Solve for w.

23. Pencil Holder You are decorating a clean soup can to make a pencil holder. You are going to glue yarn around the top and bottom of the can. The total amount y of yarn (in inches) you need is given by the equation $y = 4\pi r$, where r is the radius of the can.

a. Solve the equation for r.

b. What is the radius of the can if you need 37.68 inches of yarn? Use 3.14 for π.

$$\frac{1}{4\pi} \cdot y = 4\pi r \cdot \frac{1}{4\pi}$$

$$\frac{y}{4\pi} = r$$

$$r = \frac{y}{4\pi}$$

$$r = \frac{37.68}{4\pi}$$

$$37.68 \div (4 \cdot \pi)$$

24. **Investment** An advertisement for a bank states that you can earn \$50 interest in one year by investing in a savings account that earns 4% interest. Use the simple interest formula $I = Prt$, where I is the interest on an investment of P dollars at an interest rate r for t years.

a. Which variable should you solve for to find the amount of money you need to invest to earn the \$50 in interest?

b. Solve the simple interest equation for the variable you identified in part (a).

c. How much money do you need to invest?

$$\frac{1}{RT} \cdot I = PRT \cdot \frac{1}{RT}$$

$$\frac{I}{RT} = P$$

$$P = \frac{I}{RT}$$

$$P = \frac{50}{(.04)(1)}$$

$$P = \$1250$$

TWO OR MORE VARIABLES An equation in two variables, such as $3x + 2y = 8$, or a formula in two or more variables, such as $A = \frac{1}{2}bh$, can be rewritten so that one variable is a function of the other variable(s).

EXAMPLE 2 Rewrite an equation

Write $3x + 2y = 8$ so that y is a function of x.

$3x + 2y = 8$ Write original equation.

$2y = 8 - 3x$ Subtract $3x$ from each side.

$y = 4 - \frac{3}{2}x$ Divide each side by 2.

Practice Level A

1. no **2.** no **3.** yes **4.** $y = 3 - 10x$
5. $y = 4x + 13$ **6.** $y = 4 - 8x$ **7.** $y = 7 - 2x$
8. $y = 3x + 9$ **9.** $y = 9x - 8$ **10.** $y = 3x - 4$

11. $y = \frac{2}{3}x - 2$ **12.** $y = 6 - 2x$
13. $y = 8 - \frac{5}{2}x$ **14.** $y = -\frac{7}{3}x - 6$
15. $y = x - 1$ **16.** $C = R - P$ **17.** $m = \frac{F}{a}$
18. $R = \frac{E}{I}$ **19.** $x = \frac{c + by}{a}$ **20.** $r = \frac{C}{2\pi}$
21. $B = \frac{3V}{h}$ **22.** $w = \frac{P - 2\ell}{2}$ **23. a.** $r = \frac{y}{4\pi}$
b. 3 in. **24. a.** P **b.** $P = \frac{I}{rt}$ **c.** \$1250

Name ______________________ Date ____________

LESSON 2.8

Practice B

For use with the lesson "Rewrite Equations and Formulas"

Write the equation in function form.

1. $4x + y = -10$
2. $6 - y = 17x$
3. $y - 3x - 11 = 0$
4. $2x + 2y = 8$
5. $6x - 3y = 12$
6. $16 - 8y = 4x$
7. $5x - 7y = 14$
8. $9y - 4x - 9 = 0$
9. $15 + 3y = -24x$
10. $4 + 6y = 12x - 2$
11. $4 - 10y = 22 - 6x$
12. $8x - 2y - 5 = 11$

Solve the literal equation.

13. Solve $R = R_1 + R_2$ for R_2.
14. Solve $I = Prt$ for r.
15. Solve $C = \frac{Q}{V}$ for V.
16. Solve $y = mx + b$ for m.

Solve the formula for the indicated variable.

17. Area of a trapezoid: $A = \frac{h}{2}(b_1 + b_2)$. Solve for h.
18. Area of a rhombus: $A = \frac{1}{2}d_1d_2$. Solve for d_1.

19. Guitar Practice You practice playing your guitar every day. You spend 15 minutes practicing chords and the rest of the time practicing a new song. So the total number of minutes y you practice for the week is given by $y = 7(15 + x)$, where x is the number of minutes you spend on practicing a new song.

a. Solve the equation for x.

b. How many minutes did you spend on a new song if you practiced 210 minutes last week? 245 minutes? 315 minutes?

20. Discounts Solve for r in the formula $S = L - rL$ where S is the sale price, L is the list price, and r is the discount rate.

a. An item with a list price of \$128 goes on sale for \$51.20. Find the discount rate.

b. An item with a list price of \$56.80 goes on sale for \$36.92. Find the discount rate.

21. Cookbook You bought a cookbook while on a recent trip overseas. All of the oven temperatures are in degrees Celsius and the only formula you can remember for temperature is how to convert Fahrenheit to Celsius: $C = \frac{5}{9}(F - 32)$.

a. Solve the equation for F.

b. A recipe tells you to bake a pie in the oven at 149°C. What is this temperature in degrees Fahrenheit? Round your answer to the nearest whole degree.

Practice Level B

1. $y = -4x - 10$ **2.** $y = 6 - 17x$

3. $y = 3x + 11$ **4.** $y = 4 - x$ **5.** $y = 2x - 4$

6. $y = 2 - \frac{1}{2}x$ **7.** $y = \frac{5}{7}x - 2$ **8.** $y = \frac{4}{9}x + 1$

9. $y = -8x - 5$ **10.** $y = 2x - 1$

11. $y = \frac{3}{5}x - \frac{9}{5}$ **12.** $y = 4x - 8$

13. $R_2 = R - R_1$ **14.** $r = \frac{I}{Pt}$ **15.** $V = \frac{Q}{C}$

16. $m = \frac{y - b}{x}$ **17.** $h = \frac{2A}{b_1 + b_2}$ **18.** $d_1 = \frac{2A}{d_2}$

19. a. $x = \frac{y}{7} - 15$ **b.** 15 min; 20 min; 30 min

20. $r = \frac{L - S}{L}$; **a.** 60% **b.** 35%

21. a. $F = \frac{9}{5}C + 32$ **b.** about 300°F

EXAMPLE 1 Solve a literal equation

Solve $ax + b = c$ for x. Then use the solution to solve $2x + 5 = 11$.

Solution

STEP 1 Solve $ax + b = c$ for x.

$ax + b = c$ Write original equation.

$ax = c - b$ Subtract b from each side.

$x = \frac{c - b}{a}$ Assume $a \neq 0$. Divide each side by a.

STEP 2 Use the solution to solve $2x + 5 = 11$.

$x = \frac{c - b}{a}$ Solution of literal equation

$= \frac{11 - 5}{2}$ Substitute 2 for a, 5 for b, and 11 for c.

$= 3$ Simplify.

▶ The solution of $2x + 5 = 11$ is 3.

VARIABLES IN DENOMINATORS In Example 1, you must assume that $a \neq 0$ in order to divide by a. In general, if you have to divide by a variable when solving a literal equation, you should assume that the variable does not equal 0.

✓ GUIDED PRACTICE for Example 1

Solve the literal equation for x. Then use the solution to solve the specific equation.

1. $a - bx = c$; $12 - 5x = -3$ $x = \frac{a - c}{b}$; 3

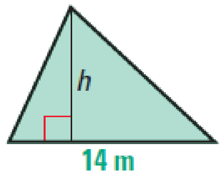
2. $ax = bx + c$; $11x = 6x + 20$ $x = \frac{c}{a - b}$; 4

EXAMPLE 3 Solve and use a geometric formula

The area A of a triangle is given by the formula $A = \frac{1}{2}bh$ where b is the base and h is the height.

a. Solve the formula for the height h.

b. Use the rewritten formula to find the height of the triangle shown, which has an area of 64.4 square meters.



Solution

a.

$A = \frac{1}{2}bh$	Write original formula.
$2A = bh$	Multiply each side by 2.
$\frac{2A}{b} = h$	Divide each side by b.

b. Substitute 64.4 for A and 14 for b in the rewritten formula.

$h = \frac{2A}{b}$	Write rewritten formula.
$= \frac{2(64.4)}{14}$	Substitute 64.4 for A and 14 for b.
$= 9.2$	Simplify.

▶ The height of the triangle is 9.2 meters.

Animated Algebra at my.hrw.com

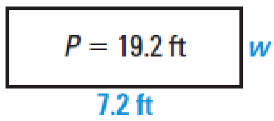
GUIDED PRACTICE for Examples 2 and 3

3. Write $5x + 4y = 20$ so that y is a function of x. $y = 5 - \frac{5}{4}x$

4. The perimeter P of a rectangle is given by the formula $P = 2\ell + 2w$ where ℓ is the length and w is the width.

   a. Solve the formula for the width w.

   b. Use the rewritten formula to find the width of the rectangle shown. 2.4 ft



EXAMPLE 4 Solve a multi-step problem

TEMPERATURE You are visiting Toronto, Canada, over the weekend. A website gives the forecast shown. Find the low temperatures for Saturday and Sunday in degrees Fahrenheit. Use the formula $C = \frac{5}{9}(F - 32)$ where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

3 Day Forecast for Toronto

Friday	Saturday	Sunday
Sunny	Sunny	Partly Cloudy
High 21°C	High 22°C	High 16°C
Low 13°C	Low 14°C	Low 10°C

Solution

STEP 1 **Rewrite** the formula. In the problem, degrees Celsius are given and degrees Fahrenheit need to be calculated. The calculations will be easier if the formula is written so that F is a function of C.

$$C = \frac{5}{9}(F - 32)$$ **Write original formula.**

$$\frac{9}{5} \cdot C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$$ **Multiply each side by $\frac{9}{5}$, the reciprocal of $\frac{5}{9}$.**

$$\frac{9}{5}C = F - 32$$ **Simplify.**

$$\frac{9}{5}C + 32 = F$$ **Add 32 to each side.**

▶ The rewritten formula is $F = \frac{9}{5}C + 32$.

STEP 2 **Find** the low temperatures for Saturday and Sunday in degrees Fahrenheit.

Saturday (low of 14°C)

$$F = \frac{9}{5}C + 32$$
$$= \frac{9}{5}(14) + 32$$
$$= 25.2 + 32$$
$$= 57.2$$

▶ The low for Saturday is 57.2°F.

Sunday (low of 10°C)

$$F = \frac{9}{5}C + 32$$
$$= \frac{9}{5}(10) + 32$$
$$= 18 + 32$$
$$= 50$$

▶ The low for Sunday is 50°F.

✓ GUIDED PRACTICE for Example 4

5. Use the information in Example 4 to find the high temperatures for Saturday and Sunday in degrees Fahrenheit. 71.6°F, 60.8°F