2.7 Solve Proportions Using Cross Products

Before You solved proportions using the multiplication property of equality.

Now You will solve proportions using cross products.

Why? So you can find the height of a scale model, as in Ex. 39.

Key Vocabulary

- **cross product**
- **scale drawing**
- **scale model**
- **scale**

In a proportion, a **cross product** is the product of the numerator of one ratio and the denominator of the other ratio. The following property involving cross products can be used to solve proportions.

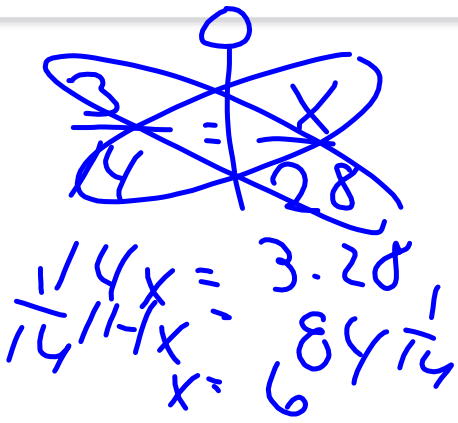
KEY CONCEPT *For Your Notebook*

Cross Products Property

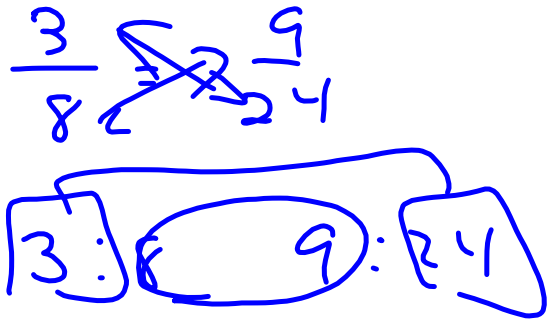
Words The cross products of a proportion are equal.

Example $\frac{3}{4} = \frac{6}{8}$ $\quad 4 \cdot 6 = 24$, $3 \cdot 8 = 24$

Algebra If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.



The proportion $\frac{3}{4} = \frac{6}{8}$ can be written as $3:4 = 6:8$. In this form, 4 and 6 are called the *means* of the proportion, and 3 and 8 are called the *extremes* of the proportion. This is why the cross products property is also called the *means-extremes property*.



LESSON 2.7

Practice A

For use with the lesson "Solve Proportions Using Cross Products"

Write the proportion in the form $a:b = c:d$. Then name the means and extremes of the proportion.

1. $\frac{4}{11} = \frac{20}{55}$

2. $\frac{18}{14} = \frac{9}{7}$

3. $\frac{25}{57} = \frac{50}{114}$

4:11 = 20:55

M: 11, 20
E: 4, 55

25:57 = 50:114

M: 57, 50
E: 25, 114

Name the cross products of the proportion.

4. $\frac{x}{15} = \frac{24}{45}$

5. $\frac{1.5}{3.5} = \frac{m}{7}$

6. $\frac{c}{70} = \frac{4}{7}$

$45x$,
$15(24)$

7. $\frac{18}{x} = \frac{90}{100}$

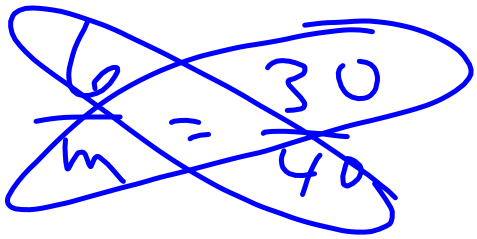
8. $\frac{3}{19} = \frac{33}{a}$

9. $\frac{6}{7} = \frac{p+4}{21}$

$3A, 19(33)$

Solve the proportion.

10. $\frac{4}{5} = \frac{12}{x}$ **11.** $\frac{6}{m} = \frac{30}{40}$ **12.** $\frac{7}{3} = \frac{56}{a}$



$30m = 6 \cdot 40$

$\frac{1}{30} 30m = 240 \cdot \frac{1}{30}$

$m = 8$

13. $\frac{70}{p} = \frac{10}{9}$

14. $\frac{15}{1} = \frac{30}{c}$

15. $\frac{1}{w} = \frac{80}{240}$

$\frac{1}{w} = \frac{80}{240}$

$80w = 1 \cdot 240$

$\frac{1}{80} 80w = 240 \frac{1}{80}$

$w = 3$

16. $\frac{16}{45} = \frac{32}{d}$

17. $\frac{88}{z} = \frac{11}{8}$

18. $\frac{60}{b} = \frac{15}{16}$

$\frac{88}{z} = \frac{11}{8}$

$88 \cdot 8 = 704$

$\frac{1}{11} \cdot 11z = 8 \cdot 8 \cdot \frac{1}{11}$

$z = 64$

$\frac{704}{11} = 64$

19. $\frac{3x}{4} = \frac{12}{16}$

20. $\frac{5y}{13} = \frac{15}{39}$

21. $\frac{2n}{14} = \frac{9}{7}$

$\frac{3x}{4} = \frac{12}{16}$ 3

$\frac{3x}{4} = \frac{3}{4}$

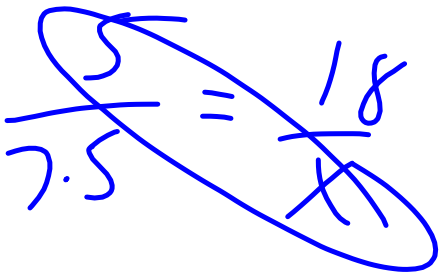
$3x \cdot 4 = 3 \cdot 4$

$12x = 12$

$x = 1$

22. Printing Pictures Your digital camera printer printed 5 pictures in 7.5 minutes. At this rate, how long will it take you to print 18 pictures?

$\frac{\text{pic}}{\text{min}}$

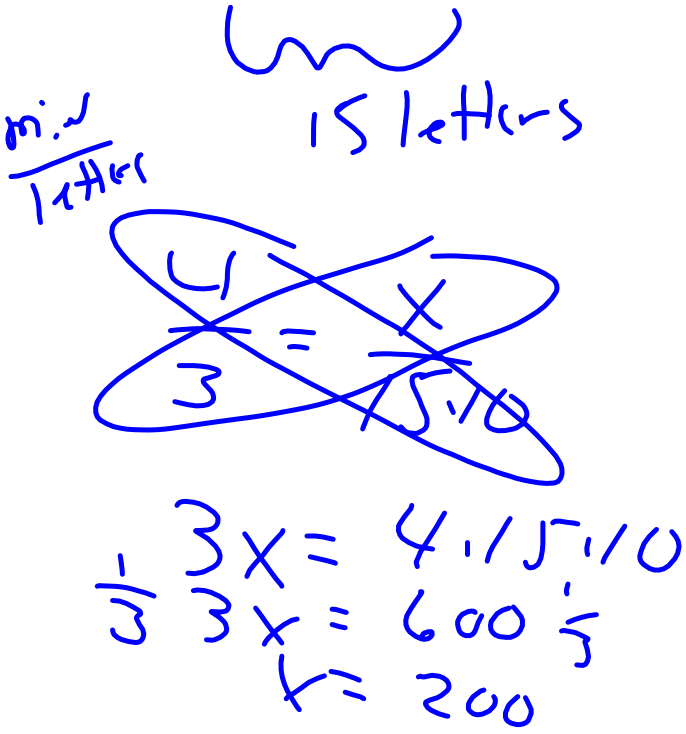


$5x = 18 \cdot 7.5$

$\frac{1}{5} 5x = 135 \frac{1}{5}$

$x = 27$

23. Stenciling You are stenciling letters on posters for an upcoming school dance. It takes you about 4 minutes to stencil 3 letters. At this rate, how long will it take you to stencil "Homecoming Dance" on 10 posters?



24. **CD Cover Art** You burned a CD of your music and then created an artistic CD label for it. Your friends liked it so much that they all asked if you would make CD labels for the CDs of their music. So far, it has taken you 6.5 hours to create 3 CD labels. How long will it take to create 4 more labels? Round your answer to the nearest tenth.

$\frac{\text{labels}}{\text{hrs}}$

$$\frac{3}{6.5} = \frac{4}{x}$$

$$\frac{1}{3}\,3x = 4 \cdot 6.5 \cdot \frac{1}{3}$$

$$x = 8.7$$

A map has a scale of 1 cm : 15 km. Use the given map distance to find the actual distance.

25. 4 cm

26. 2.5 cm

27. 7 cm

cm / km

$\frac{1}{15} = \frac{4}{x}$

$x = 4 \cdot 15$

$x = 60$

$\frac{1}{15} = \frac{7}{x}$

$x = 105$

28. Model Dam You created a model of the Grand Coulee Dam, located in the state of Washington, using a scale of 1 cm : 20 m. Your model is 15 centimeters tall. Estimate the actual height of the Grand Coulee Dam.

Practice Level A

1. 4:11 = 20 : 55; means: 11 and 20; extremes: 4 and 55 **2.** 18:14 = 9:7; means: 14 and 9;

extremes: 18 and 7 **3.** 25:57 = 50:114; means: 57 and 50; extremes: 25 and 114

4. 45x and 15(24) **5.** 7(1.5) and 3.5m

6. 7c and 70(4) **7.** 18(100) and 90x **8.** 3a and 19(33) **9.** 6(21) and 7(p + 4) **10.** $x = 15$

11. $m = 8$ **12.** $a = 24$ **13.** $p = 63$ **14.** $c = 2$

15. $w = 3$ **16.** $d = 90$ **17.** $z = 64$ **18.** $b = 64$

19. $x = 1$ **20.** $y = 1$ **21.** $n = 9$ **22.** 27 min

23. 200 min **24.** about 8.7 h **25.** 60 km

26. 37.5 km **27.** 105 km **28.** 300 m

Name _____ Date _____

LESSON 2.7

Practice B

For use with the lesson "Solve Proportions Using Cross Products"

Name the cross products of the proportion.

1. $\frac{n}{11} = \frac{40}{55}$
2. $\frac{4}{9} = \frac{1}{x}$
3. $\frac{1.8}{1.9} = \frac{b}{3.8}$
4. $\frac{a+6}{21} = \frac{4}{7}$
5. $\frac{5x}{x+1} = \frac{30}{9}$
6. $\frac{2.2}{3.3} = \frac{a-2}{a-1}$

Solve the proportion.

7. $\frac{3}{5} = \frac{21}{m}$
8. $\frac{12}{7} = \frac{60}{d}$
9. $\frac{24}{x} = \frac{48}{60}$
10. $\frac{5}{7} = \frac{3w}{21}$
11. $\frac{2w}{16} = \frac{30}{80}$
12. $\frac{2z}{24} = \frac{6}{8}$
13. $\frac{8}{9} = \frac{30+a}{45}$
14. $\frac{9-y}{44} = \frac{5}{22}$
15. $\frac{26}{15} = \frac{104}{70-w}$
16. $\frac{35}{16} = \frac{c-8}{2}$
17. $\frac{1}{9} = \frac{a}{a+24}$
18. $\frac{2}{n} = \frac{14}{n+30}$

A map has a scale of 1 in. : 38 ft. Use the given map distance to find the actual distance.

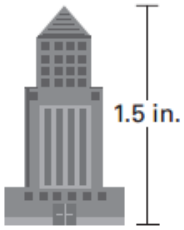
19. 5.5 in.
20. 2.25 in.
21. 1.75 in.

22. Concrete You are making up your own mix of concrete to patch a set of stairs. In order to have the proper mix, you need to mix 1 part of Portland cement with 2 parts of sand and 3 parts of gravel.

a. How many total parts are in one batch of concrete?

b. You make a mix with 4 parts of sand. How many total parts of cement, sand, and gravel are in your mix?

23. Architectural Firm An architectural firm makes a model of a science center they are building. The ratio of the model to the actual size is 1 in. : 85 ft. Estimate the height of the building if the model is 1.5 inches tall.



24. Tall Buildings You made a model of the Space Needle in Seattle, Washington, for a report on architecture in the United States. You used a scale of 1 in. : 50 ft. Your model is 12.1 inches tall. Estimate the actual height of the Space Needle.

Practice Level B

1. $55n$ and $11(40)$ **2.** $4x$ and $9(1)$ **3.** $1.8(3.8)$ and $1.9b$ **4.** $7(a + 6)$ and $21(4)$ **5.** $9(5x)$ and $30(x + 1)$ **6.** $2.2(a - 1)$ and $3.3(a - 2)$

7. $m = 35$ **8.** $d = 35$ **9.** $x = 30$ **10.** $w = 5$

11. $w = 3$ **12.** $z = 9$ **13.** $a = 10$ **14.** $y = -1$

15. $w = 10$ **16.** $c = 12.375$ **17.** $a = 3$

18. $n = 5$ **19.** 209 ft **20.** 85.5 ft **21.** 66.5 ft

22. a. 6 **b.** 12 parts **23.** 127.5 ft **24.** 605 ft

EXAMPLE 1 Use the cross products property

Solve the proportion $\frac{8}{x} = \frac{6}{15}$.

$\frac{8}{x} = \frac{6}{15}$ Write original proportion.

$8 \cdot 15 = x \cdot 6$ Cross products property

$120 = 6x$ Simplify.

$20 = x$ Divide each side by 6.

▶ The solution is 20. Check by substituting 20 for x in the original proportion.

CHECK $\frac{8}{20} \stackrel{?}{=} \frac{6}{15}$ Substitute 20 for x.

$8 \cdot 15 \stackrel{?}{=} 20 \cdot 6$ Cross products property

$120 = 120$ ✓ Simplify. Solution checks.

EXAMPLE 2 Standardized Test Practice

What is the value of x in the proportion $\frac{4}{x} = \frac{8}{x-3}$?

Ⓐ -6 Ⓑ -3 Ⓒ 3 Ⓓ 6

Solution

$\frac{4}{x} = \frac{8}{x-3}$ Write original proportion.

$4(x-3) = x \cdot 8$ Cross products property

$4x - 12 = 8x$ Simplify.

$-12 = 4x$ Subtract $4x$ from each side.

$-3 = x$ Divide each side by 4.

▶ The value of x is -3. The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 3 Write and solve a proportion

SEALS Each day, the seals at an aquarium are each fed 8 pounds of food for every 100 pounds of their body weight. A seal at the aquarium weighs 280 pounds. How much food should the seal be fed per day?

Solution

STEP 1 **Write** a proportion involving two ratios that compare the amount of food with the weight of the seal.

$$\frac{8}{100} = \frac{x}{280} \quad \begin{matrix} \leftarrow \text{amount of food} \\ \leftarrow \text{weight of seal} \end{matrix}$$

STEP 2 **Solve** the proportion.

$$\frac{8}{100} = \frac{x}{280} \qquad \text{Write proportion.}$$

$$8 \cdot 280 = 100 \cdot x \qquad \text{Cross products property}$$

$$2240 = 100x \qquad \text{Simplify.}$$

$$22.4 = x \qquad \text{Divide each side by 100.}$$

▶ A 280 pound seal should be fed 22.4 pounds of food per day.

GUIDED PRACTICE for Examples 1, 2, and 3

Solve the proportion. Check your solution.

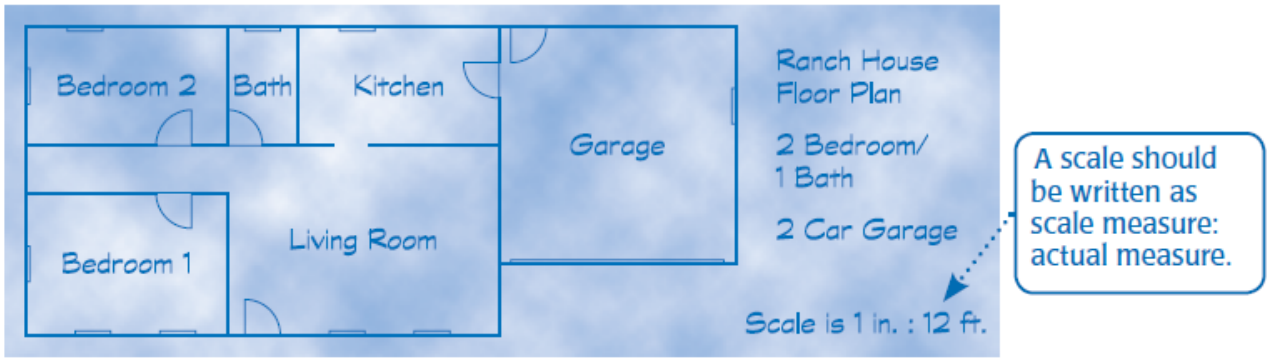
1. $\frac{4}{a} = \frac{24}{30}$ **5**

2. $\frac{3}{x} = \frac{2}{x - 6}$ **18**

3. $\frac{m}{5} = \frac{m - 6}{4}$ **30**

4. **WHAT IF?** In Example 3, suppose the seal weighs 260 pounds. How much food should the seal be fed per day? **20.8 lb**

SCALE DRAWINGS AND SCALE MODELS The floor plan below is an example of a *scale drawing*. A **scale drawing** is a two-dimensional drawing of an object in which the dimensions of the drawing are in proportion to the dimensions of the object. A **scale model** is a three-dimensional model of an object in which the dimensions of the model are in proportion to the dimensions of the object.



The **scale** of a scale drawing or scale model relates the drawing's or model's dimensions and the actual dimensions. For example, the scale 1 in. : 12 ft on the floor plan means that 1 inch in the floor plan represents an actual distance of 12 feet.

EXAMPLE 4 Use the scale on a map

MAPS Use a metric ruler and the map of Ohio to estimate the distance between Cleveland and Cincinnati.

Solution

From the map's scale, 1 centimeter represents 85 kilometers. On the map, the distance between Cleveland and Cincinnati is about 4.2 centimeters.

Write and solve a proportion to find the distance d between the cities.

$$\frac{1}{85} = \frac{4.2}{d} \quad \begin{matrix} \leftarrow \text{centimeters} \\ \leftarrow \text{kilometers} \end{matrix}$$

$1 \cdot d = 85 \cdot 4.2$ Cross products property

$d = 357$ Simplify.

▸ The actual distance between Cleveland and Cincinnati is about 357 kilometers.

GUIDED PRACTICE for Example 4

5. Use a metric ruler and the map in Example 4 to estimate the distance (in kilometers) between Columbus and Cleveland. **about 212.5 km**

6. **MODEL SHIPS** The ship model kits sold at a hobby store have a scale of 1 ft : 600 ft. A completed model of the *Queen Elizabeth II* is 1.6 feet long. Estimate the actual length of the *Queen Elizabeth II*. **about 960 ft**