2.6 Write Ratios and Proportions

Before You solved equations involving division.

Now You will find ratios and write and solve proportions.

Why? So you can find a ratio involving a contest, as in Ex. 46.

$$n+10 = \frac{5}{6}n + 7 - \frac{1}{3}n$$

$-\frac{2}{6}n$

$$\frac{1}{2}n + n + 10 = \frac{1}{2}n + 7 + \frac{1}{2}n$$

$$10 + \frac{1}{2}n + 10 = 7 + 10$$

$$\frac{2}{1} \cdot \frac{1}{2}n = 3 \cdot \frac{2}{1}$$

$$n = 6$$

$$60 + 42.95m = 57.95m$$

$$\frac{1.60}{1.5} = \frac{1.15m}{1.5}$$

$$4 = m$$

KEY CONCEPT *For Your Notebook*

Ratios

A **ratio** uses division to compare two quantities. You can write the ratio of two quantities a and b, where b is not equal to 0, in three ways.

a to b $\qquad$ $a:b$ $\qquad$ $\frac{a}{b}$

Each ratio is read "the ratio of a to b." Ratios should be written in simplest form.

PROPORTIONS A **proportion** is an equation that states that two ratios are equivalent. The general form of a proportion is given below.

READING
This proportion is read "*a* is to *b* as *c* is to *d*."

$$\frac{a}{b} = \frac{c}{d} \text{ where } b \neq 0, d \neq 0$$

If one of the numbers in a proportion is unknown, you can solve the proportion to find the unknown number. To solve a proportion with a variable in the numerator, you can use the same methods you used to solve equations.

LESSON 2.6

Practice A

For use with the lesson "Write Ratios and Proportions"

Tell whether the ratio is in simplest form. If not, write it in simplest form.

1. 15 to 27

NO

15 to 27

5 to 9

2. 7 : 18

YES

3. $\frac{11}{55}$

NO

$\frac{1}{5}$

4. 16 to 5

5. 18 : 3

6. $\frac{42}{14}$

Solve the proportion.

7. $\frac{3}{4} = \frac{x}{8}$ **8.** $\frac{1}{5} = \frac{m}{30}$ **9.** $\frac{5}{20} = \frac{a}{4}$

$\frac{2}{1} \cdot \frac{3}{4_1} = \frac{X}{8} \cdot 8$

$6 = X$

$\frac{4}{1} \cdot \frac{1}{2_1} = \frac{A}{4} \cdot 4$

$1 = A$

10. $\frac{p}{18} = \frac{4}{9}$

11. $\frac{w}{15} = \frac{2}{5}$

12. $\frac{4}{3} = \frac{y}{12}$

$15 \cdot \frac{w}{15} = \frac{2}{5} \cdot \frac{15}{1}$

$w = 6$

13. $\frac{3}{6} = \frac{z}{14}$

14. $\frac{6}{8} = \frac{c}{12}$

15. $\frac{n}{8} = \frac{9}{12}$

$8 \cdot \frac{n}{8} = \frac{9}{12} \cdot 8^{2}$

$n = 6$

Match the sentence with its proportion. Then solve the proportion.

16. 7 is to 4 as x is to 6.

17. 6 is to x as 4 is to 7.

18. x is to 7 as 4 is to 6.

A. $\frac{x}{7} = \frac{4}{6}$

B. $\frac{7}{4} = \frac{x}{6}$

C. $\frac{6}{x} = \frac{4}{7}$

19. Rainy Days Last year, it rained 18 days during a 90-day period.

a. Find the ratio in lowest terms of the number of rainy days to the total number of days in the period.

b. Find the ratio in lowest terms of the number of days it didn't rain to the total number of days in the period.

c. Find the ratio in lowest terms of the number of rainy days to the number of days it didn't rain.

A) $\frac{18}{90} = \frac{1}{5}$

B $\frac{72}{90} = \frac{4}{5}$

C $\frac{18}{72} = \frac{1}{4}$

20. Vegetable Garden You are planting the following vegetables in your vegetable garden.

Plant	Tomato	Pepper	Bean
Number of plants	8	6	4

18

a. Find the ratio of the number of bean plants to the number of tomato plants.

b. Find the ratio of the number of tomato plants to the number of bean and pepper plants.

c. Find the ratio of the number of pepper plants to the number of all plants.

A) $\frac{4}{8} = \frac{1}{2}$

B) $\frac{8}{6+4}$ $\frac{8}{10} = \frac{4}{5}$

C. $\frac{6}{18} = \frac{1}{3}$

21. **Typing** You can type 2 pages of a paper in 30 minutes. How many pages can you type in 3 hours (180 minutes)?

$$\frac{6}{1} \cdot \frac{2}{30} = \frac{n}{180} \cdot 180$$

$$12 = n$$

22. Invitations You are making party invitations by hand. It has taken you 45 minutes to make 6 cards. If you continue at this rate, how many cards can you make in 2 hours (120 minutes)?

$$\frac{6}{45} = \frac{m}{120}$$

$$16 = m$$

Practice Level A

1. no; 5 to 9 **2.** yes **3.** no; $\frac{1}{5}$ **4.** yes

5. no; 6: 1 **6.** no; $\frac{3}{1}$ **7.** $x = 6$ **8.** $m = 6$

9. $a = 1$ **10.** $p = 8$ **11.** $w = 6$ **12.** $y = 16$

13. $z = 7$ **14.** $c = 9$ **15.** $n = 6$ **16.** B; $x = \frac{21}{2}$

17. C; $x = \frac{21}{2}$ **18.** A; $x = \frac{14}{3}$ **19. a.** $\frac{1}{5}$ **b.** $\frac{4}{5}$

c. $\frac{1}{4}$ **20. a.** $\frac{1}{2}$ **b.** $\frac{4}{5}$ **c.** $\frac{1}{3}$ **21.** 12 pages

22. 16 cards

Name _____ Date _____

LESSON 2.6

Practice B

For use with the lesson "Write Ratios and Proportions"

Tell whether the ratio is in simplest form. If not, write it in simplest form.

1. 16 to 34

2. 17 : 65

3. $\frac{33}{108}$

Solve the proportion.

4. $\frac{1}{2} = \frac{p}{14}$

5. $\frac{2}{3} = \frac{x}{21}$

6. $\frac{14}{8} = \frac{y}{20}$

7. $\frac{y}{6} = \frac{15}{9}$

8. $\frac{10}{15} = \frac{m}{39}$

9. $\frac{b}{8} = \frac{50}{20}$

10. $\frac{8}{2.5} = \frac{d}{0.5}$

11. $\frac{1.4}{1.6} = \frac{z}{10}$

12. $\frac{n}{4} = \frac{0.3}{1.5}$

Write the sentence as a proportion. Then solve the proportion.

13. 5 is to 12 as x is to 48.

14. w is to 9 as 7 is to 36.

15. d is to 4 as 32 is to 56.

16. 22 is to 50 as x is to 500.

17. 10 is to 45 as b is to 225.

18. n is to 18 as 64 is to 72.

19. Books Over the summer, you read 20 books. Eight of these books were biographies.

a. Find the ratio of biographies to the total number of books.

b. Find the ratio of non-biographies to biographies.

c. Find the ratio of non-biographies to the total number of books.

20. Fitness Center The table shows the number of people attending classes at a fitness center during a recent evening.

Class	Aerobics	Spinning	Yoga
Number of people	32	28	16

a. Find the ratio of the number of people taking yoga to the number of people taking spinning class.

b. Find the ratio of the number of people taking aerobics to the total number of people taking classes.

21. Mailroom You work in the local mailroom at a college. One of your duties is to sort local mail from all of the other mail. You can sort 8 pieces of mail in 10 seconds. How many pieces of mail should you be able to sort in 45 minutes?

22. Music A music downloading website reports that nearly 5 out of every 7 songs downloaded are classified as pop music. According to this information, predict how many of the next 500 songs downloaded will be pop songs. Round your answer to the nearest whole number.

Practice Level B

1. no; 8 to 17 **2.** yes **3.** no; $\frac{11}{36}$ **4.** $p = 7$

5. $x = 14$ **6.** $y = 35$ **7.** $y = 10$ **8.** $m = 26$

9. $b = 20$ **10.** $d = 1.6$ **11.** $z = 8.75$

12. $n = 0.8$ **13.** $\frac{5}{12} = \frac{x}{48}$; $x = 20$ **14.** $\frac{w}{9} = \frac{7}{36}$; $w = \frac{7}{4}$ **15.** $\frac{d}{4} = \frac{32}{56}$; $d = \frac{16}{7}$ **16.** $\frac{22}{50} = \frac{x}{500}$; $x = 220$ **17.** $\frac{10}{45} = \frac{b}{225}$; $b = 50$ **18.** $\frac{n}{18} = \frac{64}{72}$; $n = 16$ **19. a.** $\frac{2}{5}$ **b.** $\frac{3}{2}$ **c.** $\frac{3}{5}$ **20. a.** $\frac{4}{7}$ **b.** $\frac{8}{19}$

21. 2160 pieces of mail **22.** about 357 songs

Throughout this book you have been using rates, such as 50 miles per hour. A rate is a special type of *ratio*.

Key Vocabulary

- ratio
- proportion
- simplest form

EXAMPLE 1 Write a ratio

VOLLEYBALL A volleyball team plays 14 home matches and 10 away matches.

a. Find the ratio of home matches to away matches.

b. Find the ratio of home matches to all matches.

Solution

a. $\frac{\text{home matches}}{\text{away matches}} = \frac{14}{10} = \frac{7}{5}$

b. $\frac{\text{home matches}}{\text{all matches}} = \frac{14}{14 + 10} = \frac{14}{24} = \frac{7}{12}$

GUIDED PRACTICE for Example 1

Derek and his brother decide to combine their CD collections. Derek has 44 CDs, and his brother has 52 CDs. Find the specified ratio.

1. The number of Derek's CDs to the number of his brother's CDs $\frac{11}{13}$
2. The number of Derek's CDs to the number of CDs in the entire collection $\frac{11}{24}$

EXAMPLE 2 Solve a proportion

Solve the proportion $\frac{11}{6} = \frac{x}{30}$.

$\frac{11}{6} = \frac{x}{30}$ Write original proportion.

$30 \cdot \frac{11}{6} = 30 \cdot \frac{x}{30}$ Multiply each side by 30.

$\frac{330}{6} = x$ Simplify.

$55 = x$ Divide.

GUIDED PRACTICE for Example 2

Solve the proportion. Check your solution.

3. $\frac{w}{35} = \frac{4}{7}$ 20

4. $\frac{9}{2} = \frac{m}{12}$ 54

5. $\frac{z}{54} = \frac{5}{9}$ 30

EXAMPLE 3 Solve a multi-step problem

ELEVATORS The elevator that takes passengers from the lobby of the John Hancock Center in Chicago to the observation level travels 150 feet in 5 seconds. The observation level is located on the 94th floor, at 1029 feet above the ground. Find the time it takes the elevator to travel from the lobby to the observation level.

Solution

STEP 1 **Write** a proportion involving two ratios that compare the amount of time the elevator has ascended with the distance traveled.

$$\frac{5}{150} = \frac{x}{1029} \quad \begin{matrix} \leftarrow \text{seconds} \\ \leftarrow \text{feet} \end{matrix}$$

STEP 2 **Solve** the proportion.

$$\frac{5}{150} = \frac{x}{1029}$$ Write proportion.

$$1029 \cdot \frac{5}{150} = 1029 \cdot \frac{x}{1029}$$ Multiply each side by 1029.

$$\frac{5145}{150} = x$$ Simplify.

$$34.3 = x$$ Use a calculator.

▶ The elevator travels from the lobby to the observation level in 34.3 seconds.

CHECK You can use a table to check the reasonableness of your answer.

Time (sec)	5	10	15	20	25	30	35
Distance traveled (ft)	150	300	450	600	750	900	1050

The solution, 34.3 seconds, is slightly less than 35 seconds, and 1029 feet is slightly less than 1050 feet. So, the solution is reasonable.

GUIDED PRACTICE for Example 3

6. **WHAT IF?** In Example 3, suppose the elevator travels 125 feet in 5 seconds. Find the time it will take for the elevator to travel from the lobby to the observation level. **41.16 sec**

7. **ASTRONOMY** When two full moons appear in the same month, the second full moon is called a blue moon. On average, 2 blue moons occur every 5 years. Find the number of blue moons that are likely to occur in the next 25 years. **10 blue moons**

How many tomatoes are needed to make 4 pints of salsa?

The tables below show two ways of arranging the information from the problem. In each table, x represents the number of tomatoes needed to make 4 pints of salsa. The proportions follow from the tables.

	Tomatoes	Pints
Smaller recipe	x	4
Normal recipe	30	12

Proportion: $\frac{x}{30} = \frac{4}{12}$

	Smaller recipe	Normal recipe
Tomatoes	x	30
Pints	4	12

Proportion: $\frac{x}{4} = \frac{30}{12}$