2.5 Solve Equations with Variables on Both Sides

Before You solved equations with variables on one side.

Now You will solve equations with variables on both sides.

Why? So you can find the cost of a gym membership, as in Ex. 52.

Key Vocabulary

- **identity**

Some equations have variables on both sides. To solve such equations, you can collect the variable terms on one side of the equation and the constant terms on the other side of the equation.

NUMBER OF SOLUTIONS Equations do not always have one solution. An equation that is true for all values of the variable is an **identity**. So, the solution of an identity is all real numbers. Some equations have no solution.

$x = 4$

$\begin{cases} 17 = 17 \leftarrow \mathbb{R} \\ 15 = 3 \leftarrow \text{no sol} \end{cases}$

LESSON 2.5

Practice A

For use with the lesson "Solve Equations with Variables on Both Sides"

Describe each step used in solving the equation.

1. $10x + 7 = 4x + 5$

A. $6x + 7 = 5$

B. $6x = 12$

C. $x = 2$

2. $3x + 6 = -2x + 11$

A. $5x + 6 = 11$

B. $5x = 5$

C. $x = 1$

3. $6(3x - 4) = 12$

A. $18x - 24 = 12$

B. $18x = 36$

C. $x = 2$

ADD. Prop of Equality
ADD Prop of Eq
Mult Prop Eq

4. $6(x + 3) = 5x + 8$

A. $6x + 18 = 5x + 8$

B. $x + 18 = 8$

C. $x = -10$

5. $4(x - 2) = 7x + 1$

A. $4x - 8 = 7x + 1$

B. $-8 = 3x + 1$

C. $-9 = 3x$

D. $-3 = x$

6. $2x + 2 = 4(x - 5)$

A. $2x + 2 = 4x - 20$

B. $2 = 2x - 20$

C. $22 = 2x$

D. $11 = x$

A Distr. Prop

B ADD Prop of Eq

C ADD Prop of Eq

Solve the equation and describe each step you use.

7. $6p - 3 = 4p - 1$ **8.** $10a - 2 = 7a + 4$ **9.** $5(m + 2) = 20$

Extend Page

$-4p \quad -4p$

$6p - 3 = 4p - 1$

Add Prop of Eq

$3 \quad +3 \quad = -1 + 3$

$\frac{1}{2}\,2p = 2\,\frac{1}{2}$

$p = 1$

Mult Prop of Eq.

Solve the equation, if possible.

10. $9x - 2 = 8x + 7$ **11.** $5n - 3 = 3n + 1$ **12.** $4z - 5 = 8z + 3$

$-4z \quad 4z + 5 - 8z + 3 - 4z$

$+5 \quad -3 = \quad -4z + 3 + 3$

$\frac{1}{-4} \quad -8 = \quad -4z \frac{1}{-4}$

$-2 = z$

13. $-a + 4 = a + 6$ **14.** $w + 8 = w - 3$ **15.** $2(y - 3) = y + 4$

$-w + w + 8 = w - 3 + w$

$8 = -3$

NO SOLN

16. $3(m + 2) = 8 + m$ **17.** $6 + x = 6(x - 5)$ **18.** $7(b + 3) = 7b - 4$

$3(m+2) = 8 + m$

$-m \quad -m$

$3m + 6 = 8 + m$

$\cdot 4, \quad 2m + 6 = 8$

$\frac{1}{2} \quad 2m = 2 \quad \frac{1}{2}$

$m = 1$

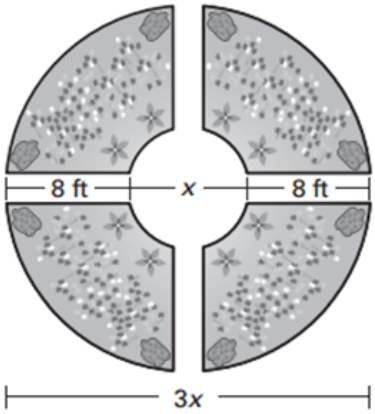
$7(b+3) = 7b+4$

$-7b \quad -7b$

$21 = 4$

NO

19. Dimensions of a Circular Flower Garden A flower garden has the shape shown. The diameter of the outer circle is three times the diameter of the inner circle. The lengths of the walkways are 8 feet long. What is the diameter of the inner circle?



$3x = (x + 8 + 8)$

$-x \quad -x$

$2x = 16$

$\frac{1}{2}\cdot 2x = 16 \cdot \frac{1}{2}$

$x = 8$

20. Distance-Rate-Time Two cars travel the same distance. The first car travels at a rate of 50 miles per hour and reaches its destination in t hours. The second car travels at a rate of 60 miles per hour and reaches its destination 1 hour earlier than the first car. How long does it take for the first car to reach its destination?

Rate of car 1	·	Time for car 1	=	Rate of car 2	·	Time for car 2

$$50 \cdot t = 60 \cdot (t-1)$$

$$-50t \quad -50t \quad = 60t + -60 \quad -50t$$

$$60 + 0 = 10t + \quad + -60 + 60$$

$$\div 10 \quad 60 = 10t \div 10$$

$$6 = t$$

Practice Level A

1. A. Subtract $4x$ from each side. **B.** Add 7 to each side. **C.** Divide each side by 6. **2. A.** Add $2x$ to each side. **B.** Subtract 6 from each side. **C.** Divide each side by 5. **3. A.** Distribute 6 to $(3x - 4)$. **B.** Add 24 to each side. **C.** Divide each side by 18. **4. A.** Distribute 6 to $(x + 3)$. **B.** Subtract $5x$ from each side. **C.** Subtract 18 from each side. **5. A.** Distribute 4 to $(x - 2)$. **B.** Subtract $4x$ from each side. **C.** Subtract 1 from each side. **D.** Divide each side by 3.

6. A. Distribute 4 to $(x - 5)$. **B.** Subtract $2x$ from each side. **C.** Add 20 to each side. **D.** Divide each side by 2.

7. $6p - 3 = 4p - 1$

$2p - 3 = -1$	Subtract $4p$ from each side.
$2p = 2$	Add 3 to each side.
$p = 1$	Divide each side by 2.

8. $10a - 2 = 7a + 4$

$3a - 2 = 4$	Subtract $7a$ from each side.
$3a = 6$	Add 2 to each side.
$a = 2$	Divide each side by 3.

9. $5(m + 2) = 20$

$5m + 10 = 20$	Distribute 5 to $(m + 2)$.
$5m = 10$	Subtract 10 from each side.
$m = 2$	Divide each side by 5.

10. $x = 9$ **11.** $n = 2$ **12.** $z = -2$ **13.** $a = -1$

14. no solution **15.** $y = 10$ **16.** $m = 1$

17. $x = 7.2$ **18.** no solution **19.** 8 ft

20. Car 1: 6 h

Name _____ Date _____

LESSON 2.5

Practice B

For use with the lesson "Solve Equations with Variables on Both Sides"

Solve the equation and describe each step you use.

1. $5x + 11 = 4x + 18$

2. $11p - 4 = 6p + 1$

3. $-6 = 2(w + 5)$

Solve the equation, if possible.

4. $15x - 8 = 14x + 13$

5. $9n - 7 = 5n + 5$

6. $4z - 15 = 4z + 11$

7. $-7a + 9 = 3a + 49$

8. $4(w + 3) = w - 15$

9. $8(y - 5) = 6y - 18$

10. $14m - 10 = 3(4 + m)$

11. $7 + x = \frac{1}{2}(4x - 2)$

12. $8b + 11 - 3b = 2b + 2$

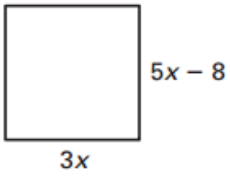
13. $10d - 6 = 4d - 15 - 3d$

14. $16p - 4 = 4(2p - 3)$

15. $0.25(8z - 4) = z + 8 - 2z$

Find the perimeter of the square.

16.



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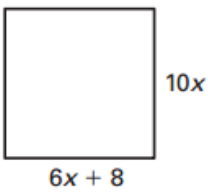

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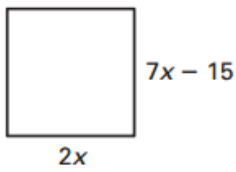

19. **Saving and Spending** Currently, you have \$80 and your sister has \$145. You decide to save \$6 of your allowance each week, while your sister decides to spend her whole allowance plus \$7 each week. How long will it be before you have as much money as your sister?

20. Botanical Gardens The membership fee for joining a gardening association is $24 per year. A local botanical garden charges members of the gardening association $3 for admission to the garden. Nonmembers of the association are charged $6. After how many visits to the garden is the total cost for members, including the membership fee, the same as the total cost for nonmembers?

21. College Enrollment Information about students' choices of majors at a small college is shown in the table. In how many years will there be 2 times as many students majoring in engineering than in business? In how many years will there be 2 times as many students majoring in engineering than in biology?

Major	Number of students enrolled in major	Average rate of change
Engineering	120	22 more students each year
Business	105	4 fewer students each year
Biology	98	6 more students each year

Practice Level B

1. $5x + 11 = 4x + 18$

$x + 11 = 18$ Subtract $4x$ from each side.

$x = 7$ Subtract 11 from each side.

2. $11p - 4 = 6p + 1$

$5p - 4 = 1$ Subtract $6p$ from each side.

$5p = 5$ Add 4 to each side.

$p = 1$ Divide each side by 5.

3. $-6 = 2(w + 5)$

$-6 = 2w + 10$ Distribute 2 to $(w + 5)$.

$-16 = 2w$ Subtract 10 from each side.

$-8 = w$ Divide each side by 2.

4. $x = 21$ **5.** $n = 3$ **6.** no solution

Lesson 2.5 Solve Equations with Variables on Both Sides, continued

7. $a = -4$ **8.** $w = -9$ **9.** $y = 11$ **10.** $m = 2$

11. $x = 8$ **12.** $b = -3$ **13.** $d = -1$

14. $p = -1$ **15.** $z = 3$ **16.** 48 **17.** 80 **18.** 24

19. 5 weeks **20.** 8 visits **21.** 3 years; 7.6 years

EXAMPLE 1 Solve an equation with variables on both sides

Solve $7 - 8x = 4x - 17$.

$7 - 8x = 4x - 17$	Write original equation.
$7 - 8x + 8x = 4x - 17 + 8x$	Add $8x$ to each side.
$7 = 12x - 17$	Simplify each side.
$24 = 12x$	Add 17 to each side.
$2 = x$	Divide each side by 12.

▶ The solution is 2. Check by substituting 2 for x in the original equation.

CHECK $7 - 8x = 4x - 17$	Write original equation.
$7 - 8(2) \stackrel{?}{=} 4(2) - 17$	Substitute 2 for x.
$-9 \stackrel{?}{=} 4(2) - 17$	Simplify left side.
$-9 = -9$ ✓	Simplify right side. Solution checks.

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EXAMPLE 2 Solve an equation with grouping symbols

Solve $9x - 5 = \frac{1}{4}(16x + 60)$.

$9x - 5 = \frac{1}{4}(16x + 60)$	Write original equation.
$9x - 5 = 4x + 15$	Distributive property
$5x - 5 = 15$	Subtract $4x$ from each side.
$5x = 20$	Add 5 to each side.
$x = 4$	Divide each side by 5.

GUIDED PRACTICE for Examples 1 and 2

Solve the equation. Check your solution.

1. $24 - 3m = 5m$ 3

2. $20 + c = 4c - 7$ 9

3. $9 - 3k = 17 - 2k$ −8

4. $5z - 2 = 2(3z - 4)$ 6

5. $3 - 4a = 5(a - 3)$ 2

6. $8y - 6 = \frac{2}{3}(6y + 15)$ 4

▶ The number of new cars sold will be twice the number of used cars sold in 4 years.

CHECK You can use a table to check your answer.

Year	0	1	2	3	4
Used cars sold	67	63	59	55	51
New cars sold	78	84	90	96	102

The number of new cars sold is twice the number of used cars sold in 4 years.

GUIDED PRACTICE for Example 3

7. **WHAT IF?** In Example 3, suppose the car dealership sold 50 new cars this year instead of 78. In how many years will the number of new cars sold be twice the number of used cars sold? **6 yr**

EXAMPLE 3 Solve a real-world problem

CAR SALES A car dealership sold 78 new cars and 67 used cars this year. The number of new cars sold by the dealership has been increasing by 6 cars each year. The number of used cars sold by the dealership has been decreasing by 4 cars each year. If these trends continue, in how many years will the number of new cars sold be twice the number of used cars sold?

Solution

Let x represent the number of years from now. So, $6x$ represents the increase in the number of new cars sold over x years and $-4x$ represents the decrease in the number of used cars sold over x years. Write a verbal model.

New cars sold this year	+	Increase in new cars sold over x years	$= 2($	Used cars sold this year	+	Decrease in used cars sold over x years	$)$
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$	
78	+	$6x$	$= 2($	67	+	$(-4x)$	$)$

$$78 + 6x = 2(67 - 4x) \quad \text{Write equation.}$$

$$78 + 6x = 134 - 8x \quad \text{Distributive property}$$

$$78 + 14x = 134 \quad \text{Add } 8x \text{ to each side.}$$

$$14x = 56 \quad \text{Subtract 78 from each side.}$$

$$x = 4 \quad \text{Divide each side by 14.}$$

▶ The number of new cars sold will be twice the number of used cars sold in 4 years.

EXAMPLE 4 Identify the number of solutions of an equation

Solve the equation, if possible.

a. $3x = 3(x + 4)$ **b.** $2x + 10 = 2(x + 5)$

Solution

a. $3x = 3(x + 4)$ Original equation

$3x = 3x + 12$ Distributive property

The equation $3x = 3x + 12$ is not true because the number $3x$ cannot be equal to 12 more than itself. So, the equation has no solution. This can be demonstrated by continuing to solve the equation.

$3x - 3x = 3x + 12 - 3x$ Subtract $3x$ from each side.

$0 = 12$ ✗ Simplify.

▶ The statement $0 = 12$ is not true, so the equation has no solution.

b. $2x + 10 = 2(x + 5)$ Original equation

$2x + 10 = 2x + 10$ Distributive property

▶ Notice that the statement $2x + 10 = 2x + 10$ is true for all values of x. So, the equation is an identity, and the solution is all real numbers.

GUIDED PRACTICE for Example 4

Solve the equation, if possible.

8. $9z + 12 = 9(z + 3)$
no solution

9. $7w + 1 = 8w + 1$
0

10. $3(2a + 2) = 2(3a + 3)$
identity