2.3 Solve Two-Step Equations

Before You solved one-step equations.

Now You will solve two-step equations.

Why? So you can find a scuba diver's depth, as in Example 4.

The equation $\frac{x}{2} + 5 = 11$ involves two operations performed on x: division by 2 and addition by 5. You typically solve such an equation by applying the inverse operations in the reverse order of the order of operations. This is shown in the table below.

Operations performed on x	Operations to isolate x
1. Divide by 2. 2. Add 5.	1. Subtract 5. 2. Multiply by 2.

LESSON 2.3

Practice A

For use with the lesson "Solve Two-Step Equations"

State the first step in solving the equation.

1. $6x + 8 = 13$

2. $9c - 3 = 17$

3. $72 = 4m + 10$

$6x + 8 = 13$

ADD -8 to both sides

4. $15 - y = 23$

5. $\frac{a}{4} + 3 = 1$

6. $\frac{n}{5} - 6 = 11$

$15 + -y = 23$

ADD -15 to both sides

Solve the equation.

7. $4n + 8 = 12$ **8.** $5y + 2 = 17$ **9.** $8x - 15 = 1$

$-8 \rightarrow 4n + 8 = 12 - 8$

$\frac{1}{4} \cdot 4n = 4 \cdot \frac{1}{4}$

$n = 1$

10. $3c - 4 = 5$ **11.** $12 = 7 - m$ **12.** $19 = 10 - b$

$-7 + 12 = 7 - m - 7$

$5 = -m$

$-5 = m$

$m = -5$

13. $\frac{p}{2} + 3 = 11$ **14.** $\frac{w}{5} + 6 = 16$ **15.** $\frac{z}{4} - 5 = 3$

$-3 + \frac{p}{2} + 3 = 11 \; -3$

$2 \cdot \frac{p}{2} = 8 \cdot 2$

$p = 16$

Match the equation with the function described.

16.	The output of a function is 8 more than 3 times the input.	**A.**	$y + 8 = 3x$
17.	The output of a function is 3 more than 8 times the input.	**B.**	$y - 8 = 3x$
18.	The output of a function is 8 less than 3 times the input.	**C.**	$y + 3 = 8x$
19.	The output of a function is 3 less than 8 times the input.	**D.**	$y - 3 = 8x$

B

D

$y + 8 = 3x + 8$

$y + 8 = 3x$

$y + 3 = 8x + 3$

$y + 3 = 8x$

LESSON 2.3

Solve the equation.

20. $3a + 5a = 16$ **21.** $4y + 7y = 22$ **22.** $5p + 2p = 28$

23. $16x - 4x = 36$ **24.** $12m - 3m = 18$ **25.** $23z - 13z = 50$

26. Band Fundraiser Your school band needs to buy new recording equipment. The equipment will cost \$3000. The band has collected \$1200 in previous fundraisers. If the band sells sandwiches at \$5 each, how many sandwiches will they need to sell to raise the remaining funds?

Cost per sandwich	•	Number of sandwiches sold	+	Money already raised	=	Cost of equipment

27. Water Skiing You and your family are taking a vacation at a lake. Your family paid a total of \$1720 to rent a boat for 5 days along with a set of water ski equipment. The total cost of the rental is the sum of the amount paid for the water ski equipment and the amount paid for the boat rental. You were charged \$45 for the equipment rental. What did you pay per day for the boat rental?

Cost per day for boat rental	•	Number of days of boat rental	+	Cost of equipment rental	=	Total amount paid

Solve the equation.

20. $3a + 5a = 16$

21. $4y + 7y = 22$

22. $5p + 2p = 28$

$\frac{1}{8}\ 8A = 16\ \cdot\frac{1}{8}$

$A = 2$

23. $16x - 4x = 36$ **24.** $12m - 3m = 18$ **25.** $23z - 13z = 50$

$16x - 4x = 36$

$\frac{1}{12} 12x = 36 \cdot \frac{1}{12}$

$x = 3$

26. Band Fundraiser Your school band needs to buy new recording equipment. The equipment will cost $3000. The band has collected $1200 in previous fundraisers. If the band sells sandwiches at $5 each, how many sandwiches will they need to sell to raise the remaining funds?

Cost per sandwich	·	Number of sandwiches sold	+	Money already raised	=	Cost of equipment

$5x + 1200 = 3000$

$-1200 \quad -1200$

$\frac{1}{5}5x = 1800 \cdot \frac{1}{5}$

$x = 360$

27. Water Skiing You and your family are taking a vacation at a lake. Your family paid a total of $1720 to rent a boat for 5 days along with a set of water ski equipment. The total cost of the rental is the sum of the amount paid for the water ski equipment and the amount paid for the boat rental. You were charged $45 for the equipment rental. What did you pay per day for the boat rental?

Cost per day for boat rental	•	Number of days of boat rental	+	Cost of equipment rental	=	Total amount paid

Practice Level A

1. Subtract 8 from each side. **2.** Add 3 to each side. **3.** Subtract 10 from each side. **4.** Subtract 15 from each side. **5.** Subtract 3 from each side.

6. Add 6 to each side. **7.** $n = 1$ **8.** $y = 3$

9. $x = 2$ **10.** $c = 3$ **11.** $m = -5$

12. $b = -9$ **13.** $p = 16$ **14.** $w = 50$

15. $z = 32$ **16.** B **17.** D **18.** A **19.** C

20. $a = 2$ **21.** $y = 2$ **22.** $p = 4$ **23.** $x = 3$

24. $m = 2$ **25.** $z = 5$ **26.** 360 sandwiches

27. \$335/day

Name _____ Date _____

LESSON 2.3

Practice B

For use with the lesson "Solve Two-Step Equations"

Solve the equation.

1. $3n + 14 = 35$

2. $7y - 10 = 11$

3. $14 = 9 - x$

4. $9c - 5 = 13$

5. $4.6 = 4m - 3.4$

6. $1.2 = 2.4 - 3b$

7. $\frac{p}{6} + 9 = 14$

8. $\frac{w}{7} - 2 = 9$

9. $\frac{z}{3} - 8 = -4$

Write an equation for the function described. Then find the input.

10. The output of a function is 5 more than 2 times the input. Find the input when the output is 17.

11. The output of a function is 10 more than 4 times the input. Find the input when the output is -26.

12. The output of a function is 14 less than 6 times the input. Find the input when the output is 22.

Solve the equation.

13. $9a + 4a = 26$

14. $14y - 6y = 48$

15. $38 = 26x - 7x$

16. $16x - 3x = -52$

17. $-9 = 11m - 8m$

18. $4.5z - 2.5z = 24$

19. **Yoga Class** A fitness center offers yoga classes for \$10 per class and sells yoga mats for \$19.95. A person paid a total of \$139.95 to the fitness center for yoga classes and a mat. Find the number of yoga classes the person took.

20. Library Books Your school has a $1200 grant to buy books and magazine subscriptions for the school library. The average cost of a magazine subscription is $30. Your school decides to spend $870 on books and the remaining amount on magazine subscriptions. How many magazine subscriptions can the school buy?

21. Walking You have already walked 5 miles of an 18-mile trail. If you walk the rest of the trail at a pace of 1 mile in 17 minutes, how many hours will it take you to finish the trail? Use the following verbal model to answer the question. Round your answer to the nearest tenth.

Walking rate (mi/min)	•	Number of minutes(min)	+	Number of miles already walked (mi)	=	Total number of miles walked (mi)

22. Swimming Pool The capacity of a small children's swimming pool is 106 gallons of water. There are currently 15 gallons of water in the pool. You are filling the pool with water at a rate of 2 gallons per minute.

a. Write an equation that gives the amount y (in gallons) of water in the pool as a function of the number x of minutes from now.

b. After how many minutes will the pool be full?

Practice Level B

1. $n = 7$ **2.** $y = 3$ **3.** $x = -5$ **4.** $c = 2$

5. $m = 2$ **6.** $b = 0.4$ **7.** $p = 30$ **8.** $w = 77$

9. $z = 12$ **10.** $y = 2x + 5$; 6 **11.** $y = 4x + 10$; -9 **12.** $y = 6x - 14$; 6 **13.** $a = 2$ **14.** $y = 6$

15. $x = 2$ **16.** $x = -4$ **17.** $m = -3$ **18.** $z = 12$

19. 12 classes **20.** 11 subscriptions **21.** about 3.7 h **22. a.** $y = 15 + 2x$ **b.** 45.5 min

Key Vocabulary

- like terms
- input
- output

EXAMPLE 1 Solve a two-step equation

Solve $\frac{x}{2} + 5 = 11$.

$\frac{x}{2} + 5 = 11$ — Write original equation.

$\frac{x}{2} + 5 - 5 = 11 - 5$ — Subtract 5 from each side.

$\frac{x}{2} = 6$ — Simplify.

$2 \cdot \frac{x}{2} = 2 \cdot 6$ — Multiply each side by 2.

$x = 12$ — Simplify.

▶ The solution is 12. Check by substituting 12 for x in the original equation.

CHECK $\frac{x}{2} + 5 = 11$ — Write original equation.

$\frac{12}{2} + 5 \stackrel{?}{=} 11$ — Substitute 12 for x.

$11 = 11$ ✓ — Simplify. Solution checks.

GUIDED PRACTICE for Example 1

Solve the equation. Check your solution.

1. $5x + 9 = 24$ 3

2. $4y - 4 = 16$ 5

3. $-1 = \frac{z}{3} - 7$ 18

EXAMPLE 2 Solve a two-step equation by combining like terms

REVIEW LIKE TERMS
You may want to review combining like terms before solving two-step equations.

Solve $7x - 4x = 21$.

$7x - 4x = 21$ — Write original equation.

$3x = 21$ — Combine like terms.

$\frac{3x}{3} = \frac{21}{3}$ — Divide each side by 3.

$x = 7$ — Simplify.

EXAMPLE 3 Find an input of a function

The output of a function is 3 less than 5 times the input. Find the input when the output is 17.

Solution

STEP 1 **Write** an equation for the function. Let x be the input and y be the output.

$y = 5x - 3$ — *y is 3 less than 5 times x.*

STEP 2 **Solve** the equation for x when $y = 17$.

$y = 5x - 3$	Write original function.
$17 = 5x - 3$	Substitute 17 for y.
$17 + 3 = 5x - 3 + 3$	Add 3 to each side.
$20 = 5x$	Simplify.
$\frac{20}{5} = \frac{5x}{5}$	Divide each side by 5.
$4 = x$	Simplify.

▶ An input of 4 produces an output of 17.

CHECK

$y = 5x - 3$	Write original function.
$17 \stackrel{?}{=} 5(4) - 3$	Substitute 17 for y and 4 for x.
$17 \stackrel{?}{=} 20 - 3$	Multiply 5 and 4.
$17 = 17$ ✓	Simplify. Solution checks.

GUIDED PRACTICE for Examples 2 and 3

Solve the equation. Check your solution.

4. $4w + 2w = 24$ 4

5. $8t - 3t = 35$ 7

6. $-16 = 5d - 9d$ 4

7. The output of a function is 5 more than -2 times the input. Find the input when the output is 11. -3

8. The output of a function is 4 less than 4 times the input. Find the input when the output is 3. $1\frac{3}{4}$

EXAMPLE 4 Solve a multi-step problem

SCUBA DIVING As a scuba diver descends into deeper water, the pressure of the water on the diver's body steadily increases.

The pressure at the surface of the water is 2117 pounds per square foot (lb/ft^2). The pressure increases at a rate of 64 pounds per square foot for each foot the diver descends. Find the depth at which a diver experiences a pressure of 8517 pounds per square foot.

Solution

STEP 1 **Write** a verbal model. Then write an equation.

Pressure at given depth (lb/ft^2)	=	Pressure at surface (lb/ft^2)	+	Rate of change of pressure (lb/ft^2 per foot of depth)	·	Diver's depth (ft)
P	=	2117	+	64	·	d

STEP 2 **Find** the depth at which the pressure is 8517 pounds per square foot.

$$P = 2117 + 64d$$ Write equation.

$$8517 = 2117 + 64d$$ Substitute 8517 for P.

$$8517 - 2117 = 2117 - 2117 + 64d$$ Subtract 2117 from each side.

$$6400 = 64d$$ Simplify.

$$\frac{6400}{64} = \frac{64d}{64}$$ Divide each side by 64.

$$100 = d$$ Simplify.

▶ A diver experiences a pressure of 8517 pounds per square foot at a depth of 100 feet.

CHECK

$$P = 2117 + 64d$$ Write original equation.

$$8517 \stackrel{?}{=} 2117 + 64(100)$$ Substitute 8517 for P and 100 for d.

$$8517 \stackrel{?}{=} 2117 + 6400$$ Multiply 64 and 100.

$8517 = 8517$ ✓ Simplify. Solution checks.

GUIDED PRACTICE for Example 4

9. **WHAT IF?** In Example 4, suppose the diver experiences a pressure of 5317 pounds per square foot. Find the diver's depth. **50 ft**

10. **JOBS** Kim has a job where she makes $8 per hour plus tips. Yesterday, Kim made $53 dollars, $13 of which was from tips. How many hours did she work? **5 h**