2.1 Find Square Roots and Compare Real Numbers

Before	You found squares of numbers and compared rational numbers.
Now	You will find square roots and compare real numbers.
Why?	So you can find side lengths of geometric shapes, as in Ex. 52.

Key Vocabulary
- **square root**
- **radicand**
- **perfect square**
- **irrational number**
- **real numbers**

Recall that the square of 4 is $4^2 = 16$ and the square of -4 is $(-4)^2 = 16$. The numbers 4 and -4 are called the *square roots* of 16. In this lesson, you will find the square roots of nonnegative numbers.

KEY CONCEPT — *For Your Notebook*

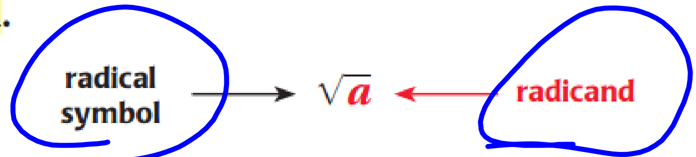
Square Root of a Number

Words If $b^2 = a$, then b is a **square root** of a.

Example $3^2 = 9$ and $(-3)^2 = 9$, so 3 and -3 are square roots of 9.

$\sqrt{100} = 10$

All positive real numbers have two square roots, a positive square root (or *principal* square root) and a negative square root. A square root is written with the radical symbol $\sqrt{}$. The number or expression inside a radical symbol is the **radicand.**



Zero has only one square root, 0. Negative real numbers do not have real square roots because the square of every real number is either positive or 0.

$\sqrt{0} = 0$

$\sqrt{-100}$ no sol

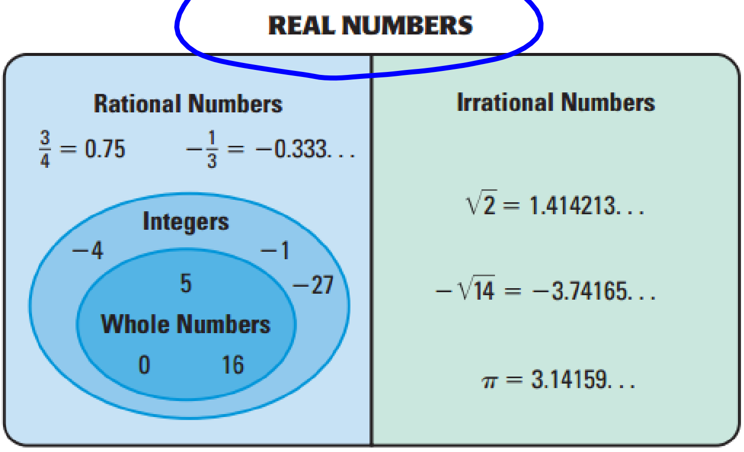
PERFECT SQUARES The square of an integer is called a **perfect square**. As shown in Example 1, the square root of a perfect square is an integer. As you will see in Example 2, you need to approximate a square root if the radicand is a whole number that is *not* a perfect square.

IRRATIONAL NUMBERS The square root of a whole number that is not a perfect square is an example of an *irrational number*. An **irrational number**, such as $\sqrt{945} = 30.74085...$, is a number that cannot be written as a quotient of two integers. The decimal form of an irrational number neither terminates nor repeats.

$\sqrt{10}$ ← Irrational

$\sqrt{9} = 3$ ✓ ← Rational

REAL NUMBERS The set of **real numbers** is the set of all rational and irrational numbers, as illustrated in the Venn diagram below. Every point on the real number line represents a real number.



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LESSON 2.1

Practice A

For use with the lesson "Find Square Roots and Compare Real Numbers"

Write the number as a power.

1. 36

36

6^2

2. 100

10^2

3. 9

Evaluate the expression.

4. $\sqrt{49}$

7

5. $-\sqrt{4}$

$-\sqrt{4}$

-2

6. $-\sqrt{25}$

7. $\sqrt{81}$

8. $-\sqrt{121}$

9. $\pm\sqrt{16}$

$\pm\sqrt{16}$

± 4

Write the greatest perfect square less than the number and the least perfect square greater than the number.

10. 13 **11.** 28 **12.** 45

Approximate the square root to the nearest integer.

13. $\sqrt{5}$

$\sqrt{4}\ \sqrt{5}\ \sqrt{9}$

$2 < \sqrt{5} < 3$

≈ 2

14. $\sqrt{19}$

15. $-\sqrt{28}$

$\sqrt{25}\ \sqrt{28}\ \sqrt{36}$

$5 \quad 6$

≈ -5

16. $-\sqrt{53}$

17. $-\sqrt{11}$

18. $\sqrt{70}$

$\sqrt{9}$ $\sqrt{11}$ $\sqrt{16}$

3 4

≈ -3

$\sqrt{70}$

$\sqrt{64}$ $\sqrt{81}$

8 ≈ 8 9

Tell whether each number in the list is a real number, a rational number, an irrational number, an integer, or a whole number. Then order the numbers from least to greatest.

19. $\sqrt{64}, -5, \sqrt{9}, 2$

20. $\sqrt{3}, 5.5, -\sqrt{16}, 0$

21. $\frac{2}{3}, \sqrt{4}, -3.6, -\sqrt{1}$

22. $-\sqrt{6}, \frac{5}{2}, 7, -4$

23. **Area Rug** You are considering buying a square area rug that has an area of 25 square feet. Find the side length of the area rug.

$\sqrt{25}$ 5

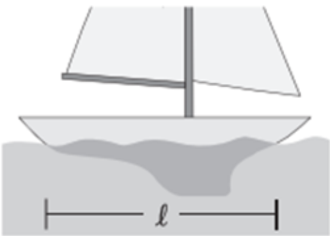
24. **Road Sign** The U.S. Department of Transportation determines the sizes of the traffic control signs that you see along the roadways. The square Alabama state route sign at the right has an area of 576 square inches. Find the side length of the sign.

$\sqrt{576}$

24

$$\begin{array}{r} 24 \\ \times\ 24 \\ \hline 96 \\ 480 \\ \hline \end{array}$$

25. Sailboat You can determine the top speed (in knots) of a sailboat using the expression $1.34\sqrt{\ell}$, where ℓ is the boat's length (in feet) where it meets the water. Find the top speed of a sailboat with a length of 10 feet at the water. Round your answer to the nearest tenth.



$1.34\sqrt{\ell}$

$1.34\sqrt{10}$

≈ 4.2

Practice Level A

1. 6^2 **2.** 10^2 **3.** 3^2 **4.** 7 **5.** -2 **6.** -5 **7.** 9 **8.** -11 **9.** ± 4 **10.** 9; 16 **11.** 25; 36 **12.** 36; 49 **13.** 2 **14.** 4 **15.** -5 **16.** -7 **17.** -3 **18.** 8

19. real number: $-5, 2, \sqrt{9}, \sqrt{64}$; rational number $-5, 2, \sqrt{9}, \sqrt{64}$; irrational number: none; integer: $-5, 2, \sqrt{9}, \sqrt{64}$; whole number: $2, \sqrt{9}, \sqrt{64}$; $-5, 2, \sqrt{9}, \sqrt{64}$ **20.** real number: $-\sqrt{16}, 0, \sqrt{3}, 5.5$; rational number: $-\sqrt{16}, 0, 5.5$; irrational number: $\sqrt{3}$; integer: $-\sqrt{16}, 0$; whole number: 0; $-\sqrt{16}, 0$ $\sqrt{3}$, 5.5 **21.** real number: $-3.6, -\sqrt{1}, \frac{2}{3}, \sqrt{4}$; rational number: $-3.6, -\sqrt{1}, \frac{2}{3}, \sqrt{4}$; irrational number: none; integer: $-\sqrt{1}, \sqrt{4}$; whole number: $\sqrt{4}$; $-3.6, -\sqrt{1}, \frac{2}{3}, \sqrt{4}$ **22.** real number: $-4, -\sqrt{6}, \frac{5}{2}, 7$; rational number: $-4, \frac{5}{2}, 7$; irrational number: $-\sqrt{6}$; integer: $-4, 7$; whole number: 7; $-4, -\sqrt{6}, \frac{5}{2}, 7$ **23.** 5 ft **24.** 24 in. **25.** about 4.2 knots

LESSON 2.1

Practice B

For use with the lesson "Find Square Roots and Compare Real Numbers"

Evaluate the expression.

1. $\pm\sqrt{81}$
2. $\pm\sqrt{25}$
3. $-\sqrt{400}$
4. $\sqrt{625}$
5. $\sqrt{4900}$
6. $\pm\sqrt{169}$

Approximate the square root to the nearest integer.

7. $-\sqrt{29}$
8. $\sqrt{108}$
9. $-\sqrt{53}$
10. $\sqrt{138}$
11. $-\sqrt{55}$
12. $\sqrt{640}$

Tell whether each number in the list is a real number, a rational number, an irrational number, an integer, or a whole number. Then order the numbers from least to greatest.

13. $-\sqrt{16}, 3.2, -\frac{3}{2}, \sqrt{9}$
14. $\sqrt{5}, -6, 2.5, -\frac{24}{5}$

Evaluate the expression for the given value of *x*.

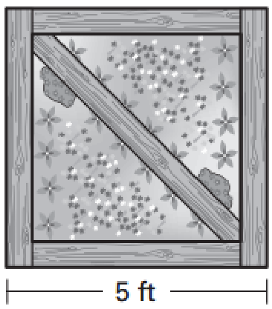
15. $14 + \sqrt{x}$ when $x = 16$
16. $\sqrt{x} - 5.5$ when $x = 4$
17. $-9 \cdot \sqrt{x}$ when $x = 25$
18. $2\sqrt{x} - 1$ when $x = 100$

19. **Park** A local park is in the shape of a square and covers an area of 3600 square feet. Find the side length of the park.

20. **Wall Poster** You are considering buying a square wall poster that has an area of 6.25 square feet. Find the side length of the wall poster.

21. **Road Sign** The U.S. Department of Transportation determines the sizes of the traffic control signs that you see along the roadways. The square Pennsylvania state route sign at the right has an area of 1296 square inches. Find the side length of the sign.

22. **Flower Bed** You are building the square flower bed shown using railroad ties. You want to place another railroad tie on the diagonal to form two triangular beds. Find the length of the diagonal by using the expression $\sqrt{2s^2}$ where s is the side length of the flower bed. Round your answer to the nearest tenth.



Practice Level B

1. ± 9 **2.** ± 5 **3.** -20 **4.** 25 **5.** 70 **6.** ± 13 **7.** -5 **8.** 10 **9.** -7 **10.** 12 **11.** -7 **12.** 25 **13.** real number: $-\sqrt{16}$, 3.2, $-\frac{3}{2}$, $\sqrt{9}$; rational number: $-\sqrt{16}$, 3.2, $-\frac{3}{2}$, $\sqrt{9}$; irrational number: none; integer: $-\sqrt{16}$, $\sqrt{9}$; whole number: $\sqrt{9}$; $-\sqrt{16}$, $-\frac{3}{2}$, $\sqrt{9}$, 3.2 **14.** real number: -6, $-\frac{24}{5}$, $\sqrt{5}$, 2.5; rational number: -6, $-\frac{24}{5}$, 2.5; irrational number: $\sqrt{5}$; integer: -6; whole number: none; -6, $-\frac{24}{5}$, $\sqrt{5}$, 2.5

15. 18 **16.** -3.5 **17.** -45 **18.** 19 **19.** 60 ft **20.** 2.5 ft **21.** 36 in. **22.** about 7.1 ft

EXAMPLE 1 Find square roots

READING
The symbol ± is read as "plus or minus" and refers to both the positive square root and the negative square root.

Evaluate the expression.

a. $\pm\sqrt{36} = \pm 6$ — The positive and negative square roots of 36 are 6 and −6.

b. $\sqrt{49} = 7$ — The positive square root of 49 is 7.

c. $-\sqrt{4} = -2$ — The negative square root of 4 is −2.

✓ GUIDED PRACTICE for Example 1

Evaluate the expression.

1. $-\sqrt{9}$ **−3**

2. $\sqrt{25}$ **5**

3. $\pm\sqrt{64}$ **±8**

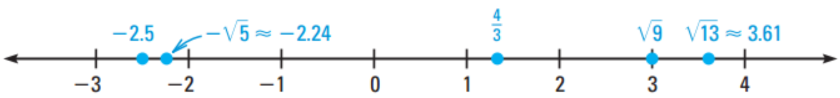
4. $-\sqrt{81}$ **−9**

EXAMPLE 4 Graph and order real numbers

Order the numbers from least to greatest: $\frac{4}{3}$, $-\sqrt{5}$, $\sqrt{13}$, -2.5, $\sqrt{9}$.

Solution

Begin by graphing the numbers on a number line.



▸ Read the numbers from left to right: -2.5, $-\sqrt{5}$, $\frac{4}{3}$, $\sqrt{9}$, $\sqrt{13}$.

GUIDED PRACTICE for Examples 3 and 4

9. Tell whether each of the following numbers is a real number, a rational number, an irrational number, an integer, or a whole number: $-\frac{9}{2}$, 5.2, 0, $\sqrt{7}$, 4.1, $-\sqrt{20}$. Then order the numbers from least to greatest. **See margin.**

EXAMPLE 2 Approximate a square root

FURNITURE The top of a folding table is a square whose area is 945 square inches. Approximate the side length of the tabletop to the nearest inch.

Solution

You need to find the side length s of the tabletop such that $s^2 = 945$. This means that s is the positive square root of 945. You can use a table to determine whether 945 is a perfect square.

Number	28	29	30	31	32
Square of number	784	841	900	961	1024

As shown in the table, 945 is *not* a perfect square. The greatest perfect square less than 945 is 900. The least perfect square greater than 945 is 961.

$900 < 945 < 961$ **Write a compound inequality that compares 945 with both 900 and 961.**

$\sqrt{900} < \sqrt{945} < \sqrt{961}$ **Take positive square root of each number.**

$30 < \sqrt{945} < 31$ **Find square root of each perfect square.**

The average of 30 and 31 is 30.5, and $(30.5)^2 = 930.25$. Because $945 > 930.25$, $\sqrt{945}$ is closer to 31 than to 30.

▶ The side length of the tabletop is about 31 inches.

✓ **GUIDED PRACTICE** for Example 2

Approximate the square root to the nearest integer.

5. $\sqrt{32}$ **6** **6.** $\sqrt{103}$ **10** **7.** $-\sqrt{48}$ **−7** **8.** $-\sqrt{350}$ **−19**

EXAMPLE 3 Classify numbers

Tell whether each of the following numbers is a real number, a rational number, an irrational number, an integer, or a whole number: $\sqrt{24}$, $\sqrt{100}$, $-\sqrt{81}$.

Number	Real number?	Rational number?	Irrational number?	Integer?	Whole number?
$\sqrt{24}$	Yes	No	Yes	No	No
$\sqrt{100}$	Yes	Yes	No	Yes	Yes
$-\sqrt{81}$	Yes	Yes	No	Yes	No

CONDITIONAL STATEMENTS A conditional statement not in if-then form can be written in that form.

EXAMPLE 5 Rewrite a conditional statement in if-then form

Rewrite the given conditional statement in if-then form. Then tell whether the statement is *true* or *false*. If it is false, give a counterexample.

Solution

a. Given: No fractions are irrational numbers.

If-then form: If a number is a fraction, then it is not an irrational number.

The statement is true.

b. Given: All real numbers are rational numbers.

If-then form: If a number is a real number, then it is a rational number.

The statement is false. For example, $\sqrt{2}$ is a real number but *not* a rational number.

GUIDED PRACTICE for Example 5

Rewrite the conditional statement in if-then form. Then tell whether the statement is *true* or *false*. If it is false, give a counterexample.

10. All square roots of perfect squares are rational numbers.
If a number is the square root of a perfect square, then it is a rational number; true.

11. All repeating decimals are irrational numbers.

12. No integers are irrational numbers.
If a number is an integer, then it is not an irrational number; true.