Section 7.3

Antidifferentiation by Parts

Homework Day 2:

#21-24

What you'll learn about

- Product Rule in Integral Form
- Solving for the Unknown Integral
- Tabular Integration
- Inverse Trigonometric and Logarithmic Functions

… and why

The Product Rule relates to derivatives as the technique of parts relates to antiderivatives.

6. $\int x^2 e^{-x}\,dx$

$u = x^2$ $\quad v = \int e^{-x} = -e^{-x}$

$du = 2x\,dx$ $\quad dv = e^{-x}\,dx$

$uv - \int v\,du$

$-x^2e^{-x} + \int e^{-x}\cdot 2x\,dx$

$u = 2x$ $\quad v = -e^{-x}$

$du = 2dx$ $\quad dv = e^{-x}dx$

$uv - \int v\,du$

$-x^2e^{-x} + 2xe^{-x} + 2\int e^{-x}dx$

$-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

$-e^{-x}(x^2 + 2x + 2) + C$

Evaluate $\int x \sin x dx$.

deriv		antideriv	
x	+	$\sin x$	
1	−	$-\cos x$	
0		$-\sin x$	←

$$-x\cos x + \sin x + C$$

EXAMPLE 6 Using Tabular Integration

Evaluate $\int x^3 \sin x \, dx$.

deriv		antideriv
x^3	+	$\sin x$
$3x^2$	−	$-\cos x$
$6x$	+	$-\sin x$
6	−	$\cos x$
0		$\sin x$

$$-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$$

$$\int x^4 e^{-x}\, dx$$

deriv		antideriv
x^4	+	e^{-x}
$4x^3$	−	$-e^{-x}$
$12x^2$	+	e^{-x}
$24x$	−	$-e^{-x}$
24	+	e^{-x}
0		$-e^{-x}$

$$-e^{-x}x^4 - 4e^{-x}x^3 - 12e^{-x}x^2 - 24e^{-x}x - 24e^{-x} + C$$

$$-e^{-x}\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right) + C$$