Section 7.3

Antidifferentiation by Parts

Homework Day 1:

#3,4,6

What you'll learn about

$\int x^3 \sin x \, dx$

- Product Rule in Integral Form
- Solving for the Unknown Integral
- Tabular Integration
- Inverse Trigonometric and Logarithmic Functions

… and why

The Product Rule relates to derivatives as the technique of parts relates to antiderivatives.

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

$uv - \int v\,du$

Evaluate $\int x \sin x\,dx$.

$u = x \qquad v = -\cos x$

$du = 1\,dx \qquad dv = \sin x\,dx$

$-x\cos x - \int -\cos x \cdot 1\,dx$

$-x\cos x + \int \cos x\,dx$

$-x\cos x + \sin x + C$

Example Repeated Use of Integra by Parts

Evaluate $\int 2x^2 e^x\, dx$.

$u = 2x^2 \quad v = e^x$

$du = 4x\,dx \quad dv = e^x\,dx$

$uv - \int v\,du$

$2x^2 e^x - \left[\int e^x \cdot 4x\,dx\right]$

$u = 4x \quad v = e^x$

$du = 4dx \quad dv = e^x dx$

$uv - \int v\,du$

$2x^2 e^x - \left[4xe^x - \int 4e^x dx\right]$

$2x^2 e^x - 4xe^x + 4e^x + C$

$2e^x(x^2 - 2x + 2) + C$

Example Solving for the Unknown Integral

Evaluate $\int e^x \sin x dx$.

$u = e^x \qquad v = -\cos x$

$du = e^x dx \qquad dv = \sin x dx$

$uv - \int v du$

$$\underbrace{\int e^x \sin x dx = -e^x \cos x}_{\downarrow} + \int \cos x e^x dx$$

$u = e^x \qquad v = \sin x$

$du = e^x dx \qquad dv = \cos x dx$

$uv - \int v du$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int \sin x e^x dx$$

$$+\int e^x \sin x dx \qquad\qquad +\int \sin x e^x dx$$

$$\frac{\cancel{2}\int e^x \sin x dx}{\cancel{2}} = \frac{-e^x \cos x + e^x \sin x}{2}$$

$$\int e^x \sin x dx = \frac{1}{2}\left(-e^x \cos x + e^x \sin x\right)$$

$$\frac{1}{2}e^x\left(-\cos x + \sin x\right)$$

Example Antidifferentiating ln *x*

Find $\int \ln x dx$.

$u = \ln x \qquad v = x$

$du = \frac{1}{x} dx \qquad dv = 1 dx$

$uv - \int v du$

$x \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx$

$x \ln x - \int 1 dx$

$\int \ln x dx =$

$x \ln x - x + C$