42. $\frac{dy}{dx} = \sqrt{y^2-4y+5}$

(0, 1) (1, $\underline{\underline{-1}}$) (-1, 1)

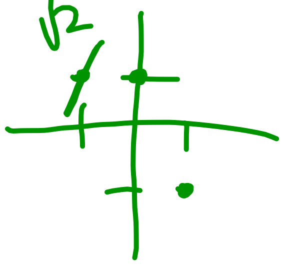
$\frac{dy}{dx} = \sqrt{1-4+5}$

$\sqrt{2}$

$\frac{dy}{dx} = \sqrt{1+4+5}$

$\sqrt{10}$

$\sqrt{2}$



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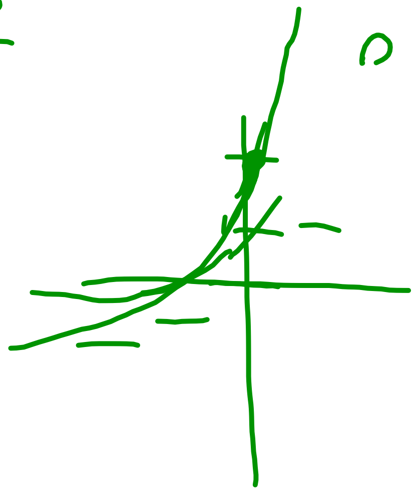

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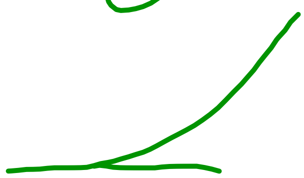

Section 7.2

Antidifferentiation by Substitution

Homework Day 1:

1-11 odd, 49,52

What you'll learn about

- Indefinite Integrals $\int$
- Leibniz Notation and Antiderivatives
- Substitution in Indefinite Integrals
- Substitution in Definite Integrals

… and why

Antidifferentiation techniques were historically crucial for applying the results of calculus.

Indefinite Integral

The family of all antiderivatives of a function $f(x)$ is the **indefinite integral of f with respect to x** and is denoted by $\int f(x)\,dx$.

If F is any function such that $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$, where C is an arbitrary constant called the **constant of integration**.

Evaluate $\int 2x - \cos x dx$.

$$\boxed{x^2 - \sin x + C}$$

Properties of Indefinite Integrals

$$\int kf(x)dx = k\int f(x)dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

Power Functions

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad \text{when } n \neq -1$$

$$\int u^{-1} du = \int \frac{1}{u} du = \ln|u| + C$$

Trigonometric Formulas

$$\int \cos u\,du = \sin u + C \qquad \int \sin u\,du = -\cos u + C$$

$$\int \sec^2 u\,du = \tan u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C \qquad \int \csc u \cot u\,du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u du = e^u + C$$

$$\int e^{2x} dx \to \frac{e^{2x}}{2} + C$$

$$\boxed{\int a^u du = \frac{a^u}{\ln a} + C}$$

$$\int 5^x dx \to \frac{5^x}{\ln 5} + C$$

$$\int \ln u\,du = u\ln u - u + C$$

$$\int \log_a u\,du = \int \frac{\ln u}{\ln a} du = \frac{u\ln u - u}{\ln a} + C$$

EXAMPLE 1 Evaluating an Indefinite Integral

Evaluate $\int (x^2 - \sin x)\, dx$.

$$\boxed{\frac{x^3}{3} + \cos x + C}$$

Let $f(x) = x^2 + 1$ and let $\boxed{u = x^3}$. Find each of the following antiderivatives in terms of x. $f(u) = u^2 + 1$

a. $\int f(x)dx = \int x^2 + 1\, dx \rightarrow \boxed{\frac{x^3}{3} + x + C}$

b. $\int f(u)du = \int u^2 + 1\, du \rightarrow \frac{u^3}{3} + u + C$

$\frac{(x^3)^3}{3} + x^3 + C = \boxed{\frac{x^9}{3} + x^3 + C}$

c. $\int f(u)dx$

$u = x^3$

$\int f(x^3)\, dx \longrightarrow \int x^6 + 1\, dx =$

$f(x) = x^2 + 1$

$f(x^3) = (x^3)^2 + 1 = x^6 + 1$

$\boxed{\frac{x^7}{7} + x + C}$

$$\int \frac{5}{x}\,dx$$

$$\frac{5}{x} = 5 \cdot \frac{1}{x}$$

$$5\int \frac{1}{x}\,dx$$

$$\boxed{5\ln|x| + C}$$

$$\int \frac{5}{x+7}$$

$$5\int \frac{1}{x+7}$$

$$5\ln|(x+7)| + C$$

$$\int 1 - 2\sin^2(x)dx$$

$\sin(2x) = 2\sin x \cos x$

$\boxed{\cos(2x)} = \cos^2 x - \sin^2 x$

$= \boxed{1 - 2\sin^2 x}$

$= 2\cos^2 x - 1$

$$\int \cos(2x)\,dx$$

$$\boxed{\frac{\sin(2x)}{2} + C}$$