Section 7.1

Slope Fields and Euler's Method

Homework:

Day 1: 3-24 by 3

Day 2: 27-48 by 3

What you'll learn about

- Differential Equations
- Slope Fields
- Euler's Method

… and why

Differential equations have been a prime motivation for the study of calculus and remain so to this day.

Differential Equation

An equation involving a derivative is called a **differential equation**. The **order of a differential equation** is the order of the highest derivative involved in the equation.

Example Solving a Differential Equation

Find all functions y that satisfy $\frac{dy}{dx} = 3x^2 + \cos x.$

$\int$ → no C

$\int$ → + C

$y = x^3 + \sin x + C$

EXAMPLE 1 Solving a Differential Equation

Find all functions y that satisfy $dy/dx = \sec^2 x + 2x + 5$.

$y = \tan x + x^2 + 5x + C$

First-order Differential Equation

If the general solution to a first-order differential equation is continuous, the only additional information needed to find a unique solution is the value of the function at a single point, called an **initial condition**. A differential equation with an initial condition is called an **initial-value problem**. It has a unique solution, called the **particular solution** to the differential equation.

Example Solving an Initial Value Problem

Find the particular solution to the equation $\frac{dy}{dx} = e^{2x} - 3x$ whose graph passes through the point $\left(1, \frac{1}{2}\right)$.

$x \quad y$

$y = \frac{e^{2x}}{2} - \frac{3x^2}{2} + C$

$\frac{1}{2} = \frac{e^2}{2} - \frac{3}{2} + C$

$+\frac{3}{2} \qquad +\frac{3}{2}$

$2 = \frac{e^2}{2} + C$

$-\frac{e^2}{2} \qquad -\frac{e^2}{2}$

$C = 2 - \frac{e^2}{2}$

$y = \frac{e^{2x}}{2} - \frac{3x^2}{2} + 2 - \frac{e^2}{2}$

Example Using the Fundamental Theorem to Solve an Initial Value Problem

Find the solution to the differential equation

$f'(x) = \cos\left(x^2\right)$ for which $f(3) = 5$.

$f(x) = y$ $\quad y = \int_3^x \cos(t^2)\,dt + 5$

EXAMPLE 4 **Using the Fundamental Theorem to Solve an Initial Value Problem**

Find the solution to the differential equation $f'(x) = e^{-x^2}$ for which $f(7) = 3$.

$$y = \int_7^x e^{-t^2}\,dt + 3$$