Evaluate the integral. Use Integration by Parts

1) $\int x e^{9x} dx$

$u = x$ $v = \dfrac{e^{9x}}{9}$

$du = 1\,dx$ $dv = e^{9x}\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$= x \cdot \frac{e^{9x}}{9} - \int \frac{e^{9x}}{9}\,dx$$

$$-\frac{1}{9}\int e^{9x}\,dx$$

$$-\frac{1}{9}\frac{e^{9x}}{9}$$

$$\frac{1}{9}x e^{9x} - \frac{1}{81}e^{9x} + C$$

$$e^{9x}\left(\frac{1}{9}x - \frac{1}{81}\right) + C$$

$$\frac{1}{9}e^{9x}\left(x - \frac{1}{9}\right) + C$$

Use tabular integration to find the antiderivative.

2) $\int (x^2 - 5x)\, e^x\, dx$

A) $\frac{1}{3}x^3 e^x - \frac{5}{2}x^2 e^x + C$

B) $e^x[x^2 - 7x - 7] + C$

C) $e^x[x^2 - 7x + 7] + C$

D) $e^x[x^2 - 5x + 5] + C$

$$\frac{deriv \quad | \quad anti}{}$$

$$x^2 - 5x \quad | \quad e^x$$
$$2x - 5 \quad | \quad e^x \quad (+)$$
$$2 \quad | \quad e^x \quad (-)$$
$$0 \quad | \quad e^x \quad (+)$$

$$e^x(x^2 - 5x) - e^x(2x - 5) + e^x(2) + C$$
$$\underbrace{-(2x-5)}$$

$$e^x\left(x^2 - 5x \underbrace{-2x + 5} \underbrace{+2}\right) + C$$

$$e^x\left(x^2 - 7x + 7\right) + C$$

Use separation of variables to solve the initial value problem.

3) $y' = 8xy$ and $y = 3$ when $x = 0$

$$dx \frac{1}{y} \frac{dy}{dx} = 8xy \cdot \frac{1}{y} dx$$

$$\frac{1}{y} dy = 8x \, dx$$

$$\frac{8x^2}{2}$$

$$\ln y = 4x^2 + C \qquad (0,3)$$

$$\ln 3 = 0 + C$$

$$C = \ln 3$$

$$\ln y = 4x^2 + \ln 3$$

$$\log_e y = 4x^2 + \ln 3$$

$$e^{4x^2 + \ln 3} = y$$

$$y = e^{4x^2} \cdot e^{\ln 3}$$

$$\underbrace{e^{\log_e 3}}$$

$$\boxed{y = 3 e^{4x^2}}$$

Find the solution of the differential equation $dy/dt = ky$, k a constant, that satisfies the given conditions.

4) $y(0) = 1710$, $k = -2.5$ 4)

initial value

$(0, 1710)$

$A = Pe^{rt}$

$y = y_0 e^{kt}$

$k = -2.5$

$$y = 1710 e^{-2.5t}$$

$$\frac{dy}{dt} = ky$$

$$\frac{1}{y} dy = k \, dt$$

Solve the problem.

5) Find the amount of time required for a $19,000 investment to double if the annual interest rate r is 5.2% and interest is compounded continuously. Round your answer to the nearest hundredth of a year.

     A) 189.47 years   B) 13.33 years   C) 202.80 years   D) 1.89 years

$$A = Pe^{rt}$$

$$\frac{38000}{19000} = \frac{19000 e^{.052t}}{19000}$$

$$2 = e^{.052t}$$

$$\log_e 2 = .052t$$

$$\frac{\ln(2)}{.052} = \frac{.052t}{.052}$$

$$t = 13.33 \text{ yrs.}$$

6) A bacterial culture has an initial population of (10,000.) If its population declines to 6000 in 6 hours, what will it be at the end of 8 hours? Assume that the population decreases according to the exponential model.

$$P = 10000 \qquad \binom{6, 6000}{t \quad A}$$

$$A = Pe^{rt}$$

$$\frac{6000}{10000} = \frac{10000 e^{r(6)}}{10000}$$

$$.6 = e^{6r}$$

$$\frac{\ln(.6)}{6} = \frac{6r}{6} \cdots$$

$$r = -.08513$$

$$t = 8\,hrs \qquad A = Pe^{rt}$$

$$A = 10000 e^{-.085\ldots(8)}$$

$$A = 5060.59594$$

$$\boxed{5061 \; bact}$$

7) How long will it take a sample of radioactive substance to decay to half of its original amount, if it decays according to the function $A(t) = 350e^{-.102t}$, where t is the time in years? Round your answer to the nearest hundredth year.

$$\frac{175}{350} = \frac{350\, e^{-.102t}}{350}$$

$$\frac{1}{2} = e^{-.102t}$$

$$\frac{\ln \frac{1}{2}}{-.102} = \frac{-.102\, t}{-.102}$$

$$t =$$

$$t = \frac{\ln 2}{K}$$

$$t = \frac{\ln 2}{.102}$$

$$t = 6.7955$$

$$\boxed{t = 6.80 \text{ yrs}}$$

Use Newton's Law of Cooling to solve the problem.

8) A dish of lasagna baked at 350°F is taken out of the oven into a kitchen that is 69°F. After 5 minutes, the temperature of the lasagna is 304.9°F. What will its temperature be 15 minutes after it was taken out of the oven? Round your answer to the nearest degree.