

# Quiz Review 2.3 to 2.5

1. For each of the following be able to...
  - a. write the roots & identify any multiplicities
  - b. determine the degree
  - c. determine if the leading coefficient is +/-
  - d. match the graph

$$f(x) = (x-3)^2(x+1)(x-5)$$

$d: 4$   
 $z: 3$   
 $mult: (2)$   
 $-1 mult: (1)$   
 $5 mult: (1)$

$$g(x) = -2x(x-3)^2$$

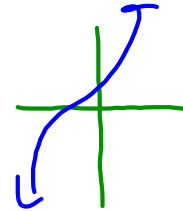
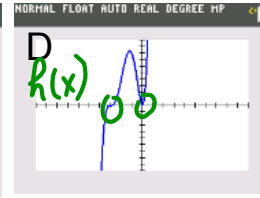
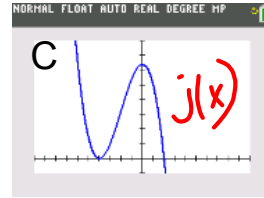
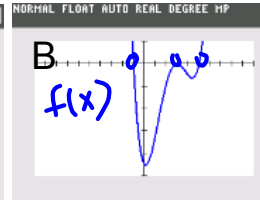
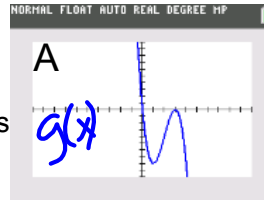
$deg 3$   
 $z: 0 mult: 1$   
 $3 mult: 2$

$$j(x) = -2(x-2)(x+4)^2$$

$deg 3$   
 $z: 2 mult: 1$   
 $-4 mult: 2$

$$h(x) = x^2(x+2)^3$$

$deg 4$   
 $z: 0 mult: 2$   
 $-1 mult: 3$



2. Use synthetic division to find a quotient and remainder when  $f(x)$  is divided by  $d(x)$ .

$$f(x) = x^3 - 5x^2 + 3x - 2$$

$$d(x) = x + 1$$

$$\begin{array}{r|rrrrr}
 x+1=0 & & & & & \\
 x=-1 & -1 & 1 & -5 & 3 & -2 \\
 & & -1 & 6 & -9 & \\
 \hline
 & 1 & -6 & 9 & -11 & 
 \end{array}$$

$q(x): x^2 - 6x + 9$   
 $r(x): -11$

3. Use long division to find a quotient and remainder when  $f(x)$  is divided by  $d(x)$ .

$$f(x) = x^4 - 3x^3 + 6x^2 - 3x + 5 \quad d(x) = x^2 + 1$$

$$\begin{array}{r} x^2 + 1 \overline{) x^4 - 3x^3 + 6x^2 - 3x + 5} \\ \underline{x^4 + x^2} \phantom{- 3x + 5} \\ -3x^3 + 5x^2 - 3x + 5 \\ \underline{-3x^3 + 3x} \phantom{+ 5} \\ 5x^2 + 2x + 5 \\ \underline{5x^2 + 5x} \\ -3x + 5 \end{array}$$

$q: x^2 + 1$   
 $r: 0$

Given  $f(x) = 3x^3 + 4x^2 - 5x - 2$ , determine which of the following are factors of  $f(x)$ .

a.  $x + 4$

b.  $x - 1$

c.  $3x + 1$

d.  $2x - 5$

① long division: Remainder = 0  
② synthetic div: Remainder = 0  
③ substitution:  $f(x) = 0$

①  $x + 4 \overline{) 3x^3 + 4x^2 - 5x - 2}$   $\begin{array}{r} -4 \end{array} \begin{array}{r} 3 \end{array} \begin{array}{r} 4 \end{array} \begin{array}{r} -5 \end{array} \begin{array}{r} -2 \end{array}$

②  $\therefore \text{no}$   $\begin{array}{r} -110 \end{array}$   $\begin{array}{r} -110 \end{array}$   
③  $f(-4) = 3(-4)^3 + 4(-4)^2 - 5(-4) - 2 = -110$  no

b)  $x = 1$   $f(1) = 0$  yes

c)  $x = -\frac{1}{3}$   $f(-\frac{1}{3}) = 0$  yes

d)  $x = \frac{5}{2}$   $f(\frac{5}{2}) = 57.375$  no

5. Use the Rational Zeros Theorem to write list of potential rational zeros.

$f(x) = 3x^4 - 2x^3 - 4x^2 - 5x - 9$

p: 1, 3, 9

q: 1, 3

$\frac{p}{q}$

$\frac{1}{1}, \frac{1}{3}, \frac{3}{1}, \frac{3}{3}, \frac{9}{1}, \frac{9}{3}$

$\pm 1, \frac{1}{3}, 3, 9$

6. Given  $f(-3) = 0$ , find all remaining zeros of  $f(x) = x^3 + x^2 - 8x - 6$

$$\begin{array}{r} -3 \overline{) 1 \quad 1 \quad -8 \quad -6} \\ \underline{-3 \quad 6 \quad 6} \\ 1 \quad -2 \quad -2 \quad 0 \end{array}$$

$$x^2 - 2x - 2 = 0$$

$$QF: \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{2 \pm \sqrt{2^2 - 4(1)(-2)}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

Solutions:  $-3, 1 \pm \sqrt{3}$

7. Given  $3i$  is a zero of  $f(x) = 3x^4 - 14x^3 + 22x^2 - 126x - 45$ , find all remaining zeros.

$$\begin{array}{r} 3i \quad -3i \\ (x-3i)(x+3i) \quad x^2+9 \overline{) 3x^4-14x^3+22x^2-126x-45} \\ \underline{3x^4 \quad +27x^2} \\ -14x^3-5x^2-126x-45 \\ \underline{-14x^3 \quad -49x} \\ -5x^2-45 \\ \underline{-5x^2 \quad -45} \\ 0 \end{array}$$

$$3x^2 - 14x - 5 = 0$$

$$(3x+1)(x-5) = 0$$

$$3x+1=0 \quad x-5=0$$

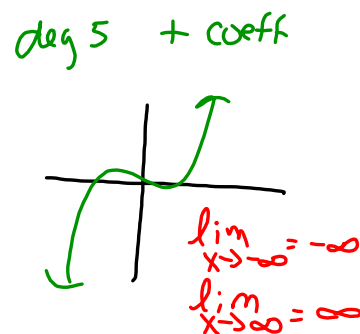
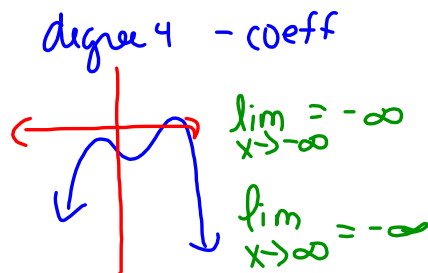
$$x=-\frac{1}{3} \quad x=5$$

Zeros:  $x = 3i, -3i, -\frac{1}{3}, 5$

8. Determine the end behavior of the following functions

$$f(x) = -17x^4 - 3x^3 + 6x^2 + 10x + 4$$

$$g(x) = 2x^5 + 3x^4 - 10x^3 + 4$$



9. Determine a polynomial with real coefficients that has roots  $-3, -\frac{3}{4}, 2$  and  $2 + 3i$

Leave it as a linear factorization of the function.

zeros  $\rightarrow$  factors

$$P(x) = (x+3)\left(x+\frac{3}{4}\right)(x-2)(x-(2+3i))(x-(2-3i))$$

$$\downarrow$$

$$(4x+3)$$