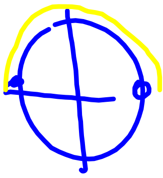
The function v(t) is the velocity in m/sec of a particle moving along the x-axis. Determine when the particle is moving to the right, to the left, and stopped.

1) $v(t) = e^{\cos t} \sin t$, $0 \le t \le 2\pi$ 1) ______________

$v(t) = e^{\cos t} \sin t$ $e^{\cos t} > 0$ always

Stopped $t = 0$ $t = \pi$ $t = 2\pi$



right $(0, \pi)$

left $(\pi, 2\pi)$

Solve the problem.

2) The velocity in m/sec of a particle moving along the x-axis is given by the function $v(t) = 2\cos^2 t \sin t$, $0 \le t \le \pi$
Find the particle's displacement for the given time interval. integral

$$-\int_0^{\pi} 2\cos^2 t \,\overset{(-)}{\sin t}\, dt \qquad u = \cos t \quad du = -\sin t\, dt$$

$$-2\int_1^{-1} u^2\, du \qquad -2\left[\frac{1}{3}u^3\right]_1^{-1}$$

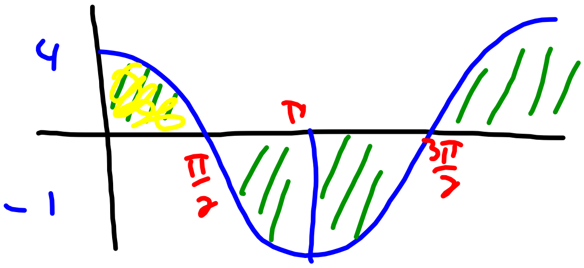
$$-\frac{2}{3}\left[u^3\right]_1^{-1}$$

$$-\frac{2}{3}\left[-1 - 1\right]$$

$$-\frac{2}{3}(-2) = \boxed{\frac{4}{3}}$$

The function v(t) is the velocity in m/sec of a particle moving along the x-axis. Find the total distance traveled by the particle.

3) $v(t) = 4\cos t,\ 0 \le t \le 2\pi$ 3) 16



$$4\int_0^{\frac{\pi}{2}} 4\cos t\,dt$$

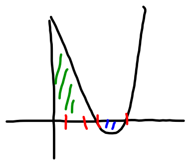
Use symmetry
4 regions of
equal area

$$16\left[\sin t\right]_0^{\frac{\pi}{2}}$$

$$16\left(\sin\tfrac{\pi}{2} - \sin 0\right)$$

$$16(1-0) = 16$$

4) $v(t) = t^2 - 7t + 12,\ 0 \le t \le 4$ $(t-3)(t-4)$



$$\int_0^3 t^2 - 7t + 12\,dt + \left|\int_3^4 t^2 - 7t + 12\,dt\right|$$

$$\tfrac{1}{3}t^3 - \tfrac{7}{2}t^2 + 12t\Big|_0^3 + \left|\tfrac{1}{3}t^3 - \tfrac{7}{2}t^2 + 12t\Big|_3^4\right|$$

$9 - \frac{63}{2} + 36$

$45 - \frac{63}{2}$

$\frac{90}{2} - \frac{63}{2} = \boxed{\frac{27}{2}}$

$\frac{64}{3} - 56 + 48 - \frac{27}{2}$

$\frac{64}{3} - 8 - \frac{27}{2}$

$\frac{64}{3} - \frac{24}{3} - \frac{27}{2}$

$\frac{40}{3} - \frac{27}{2}$

$\frac{80}{6} - \frac{81}{6}$

$= \left|-\frac{1}{6}\right|$

$\frac{27}{2} + \frac{1}{6}$

$\frac{81}{6} + \frac{1}{6} = \frac{82}{6}$

$\boxed{= \frac{41}{3}}$

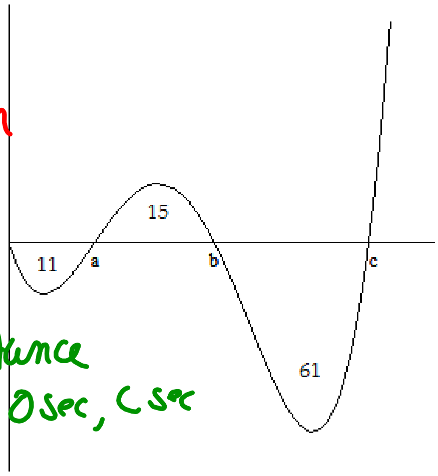
Solve the problem.

5) A particle moves along the x-axis (units in cm). Its initial position at t = 0 sec is x(0) = 14. The figure shows the graph of the particle's velocity v(t). The numbers are the areas of the enclosed regions. 5)

b) position

14 − 57

−43



c) total distance traveled 0 sec, c sec

−11 − 61 = −72

+ 15

a) −57 left 57

What is the particle's displacement between t = 0 and t = c?

11 + 15 + 61 = 87 cm

6) A surveyor measured the length of a piece of land at 100-ft intervals (x), as shown in the table. Use the Trapezoidal Rule to estimate the area of the piece of land in square feet. 6) ______________

x	Length (ft)
0	45
100	55
200	75
300	50
400	45

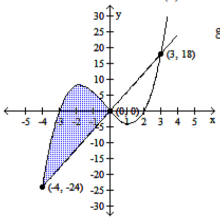
100(50 + 65 + 62.5 + 47.5)

22500 ft^2

11
47.5
62.5
65
50
225.0

Find the area of the shaded region.

7) $f(x) = x^3 + x^2 - 6x$ $g(x) = 6x$



$\int_{-4}^{0}(x^3+x^2-6x-6x)\,dx$

$\int_{-4}^{0}(x^3+x^2-12x)\,dx$

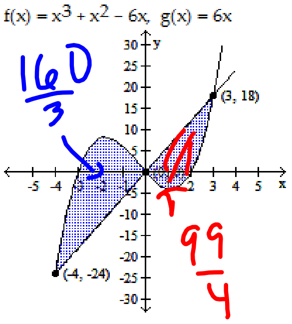
$\frac{1}{4}x^4+\frac{1}{3}x^3-6x^2\Big|_{-4}^{0}$

$0-\left[64-\frac{64}{3}-96\right]$

$0-\left[-32-\frac{64}{3}\right]$

$0-\left[-\frac{96}{3}-\frac{64}{3}\right]=\boxed{\frac{160}{3}}$

8) $f(x) = x^3 + x^2 - 6x,\ g(x) = 6x$



$\int_{0}^{3}\left[6x-(x^3+x^2-6x)\right]dx$

$\int_{0}^{3}\left[-x^3-x^2+12x\right]dx$

$-\frac{1}{4}x^4-\frac{1}{3}x^3+6x^2\Big|_{0}^{3}$

$-\frac{81}{4}-9+54-(0)$

$-\frac{81}{4}-\frac{36}{4}+\frac{216}{4}=\frac{99}{4}$

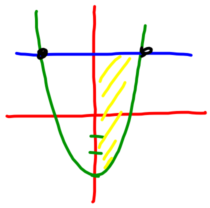
$\frac{160}{3}+\frac{99}{4}$

$\frac{640}{12}+\frac{297}{12}=\frac{937}{12}$

total area

Find the area of the regions enclosed by the lines and curves.

9) $y = x^2 - 3$ and $y = 13$



$x^2 - 3 = 13$

$x^2 = 16$

$x = \pm 4$

$\int 13 - (x^2 - 3)\,dx$

$2\int_0^4 (-x^2 + 16)\,dx$

$2\left[-\frac{1}{3}x^3 + 16x\right]_0^4$

$2\left[-\frac{64}{3} + 64\right]$

$2\left[-\frac{64}{3} + \frac{192}{3}\right]$

$2\left[\frac{128}{3}\right]$

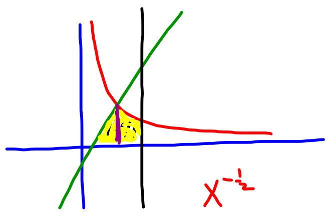
$\frac{64}{3} \cdot 2$

final answer ← $\boxed{\frac{256}{3}}$

Find the area enclosed by the given curves.

10) Find the area of the region in the first quadrant bounded by the line $y = 8x$, the line $x = 1$, the curve $y = \frac{1}{\sqrt{x}}$, and the x-axis.

10) $\frac{5}{4}$



$x^{-\frac{1}{2}}$

$8x = \frac{1}{\sqrt{x}}$

$64x^2 = \frac{1}{x}$

$64x^3 = 1$

$x^3 = \frac{1}{64}$

$x = \frac{1}{4}$

$\int_0^{\frac{1}{4}} 8x\,dx + \int_{\frac{1}{4}}^1 \frac{1}{\sqrt{x}}\,dx$

$4x^2\Big|_0^{\frac{1}{4}} + 2x^{\frac{1}{2}}\Big|_{\frac{1}{4}}^1$

$4\left(\frac{1}{16}\right) + \left[2 - 2\left(\frac{1}{2}\right)\right]$

$\frac{1}{4} + [2-1]$ 1

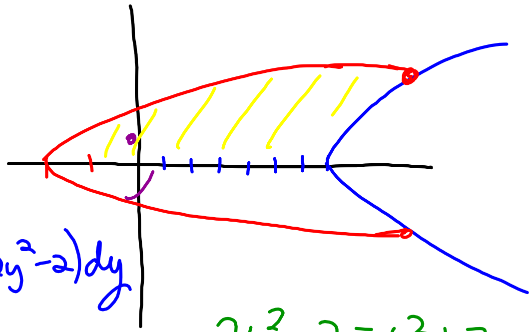
$= \boxed{\frac{5}{4}}$

Find the area of the regions enclosed by the lines and curves.

11) $x = 2y^2 - 2, \; x - y^2 = 7$

$x = 2y^2 - 2$

$x = y^2 + 7$



$\int y^2 + 7 - (2y^2 - 2)\,dy$

$2y^2 - 2 = y^2 + 7$

$y^2 = 9$

$y = \pm 3$

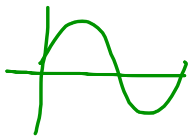
$2\int_0^3 (-y^2 + 9)\,dy$

$2\left[-\frac{1}{3}y^3 + 9y\right]_0^3$

$2[-9 + 27] \quad 2[18] = 36$

Find the area of the region(s) enclosed by the given curves.

12) $y = -4\sin x, \; y = \sin 2x, \; 0 \le x \le \pi$



$-4\sin x = 2\sin x\cos x$

$2\sin x\cos x + 4\sin x = 0$

$2\sin x(\cos x + 2) = 0$

$\sin x = 0 \quad \cancel{\cos x = -2}$

$[0, \pi]$

$\int_0^\pi \sin 2x - (-4\sin x)\,dx$

$+$

$-\frac{1}{2}\cos 2x - 4\cos x \Big|_0^\pi$

$-\frac{1}{2}\cos 2\pi - 4\cos\pi$

$- \left(-\frac{1}{2}\cos 0 - 4\cos 0\right)$

$\left[-\frac{1}{2} + 4\right] - \left(-4\frac{1}{2}\right)$

$3\frac{1}{2} + 4\frac{1}{2} \; \boxed{= 8}$