Chapter 19

Unfamiliar functions

A PROPERTIES OF FUNCTIONS

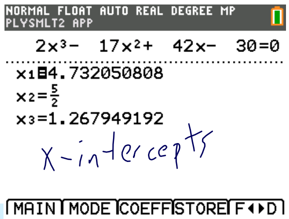
Real world situations are not always modelled by simple linear or quadratic functions that we are familiar with. However, we can use technology to help us graph and investigate the key features of an unfamiliar function.

The main features we are interested in are:

- the axes intercepts where the graph cuts the x and y-axes
- turning points (local maxima and local minima)
- the domain and range
- domain → values of x where the function does not exist
- the presence of asymptotes, or lines that the graph approaches.

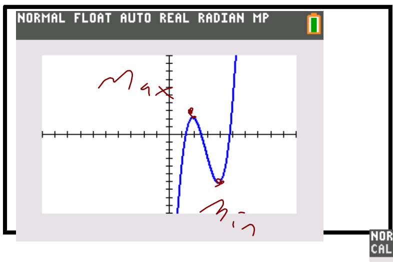
When graphing a function using technology, it is important to start with a large viewing window. This ensures we do not miss any key features of the function.

Example 1 Consider the function $y = 2x^3 - 17x^2 + 42x - 30$.



x-intercepts

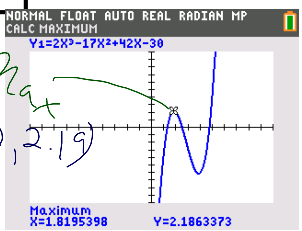
a Use technology to help sketch the function.



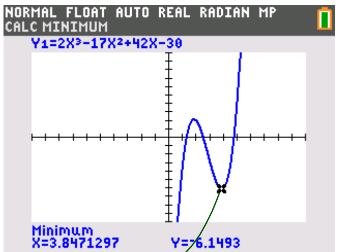
b Find the axes intercepts.

y int: let x=0 (0,-30)

c Find the coordinates and nature of any turning points.



max (1.82, 2.19)



min (3.85, -6.15)

d Discuss the behaviour of the function as $x \to \infty$ and $x \to -\infty$.

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

Example 2 Sketch the graph of $f(x) = x^3 - 3x^2 + 4x + 2$ on the domain $-2 \leqslant x \leqslant 3$.

1) Find the endpoints ____________________

2) Find the x intercept(s) ________________

3) Find the y intercept _________________

4) Find the turning points of the function

____

Sketch the curve

B ASYMPTOTES

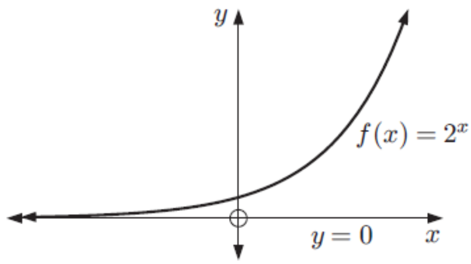
In this course we consider asymptotes which are **horizontal** or **vertical**.

An **asymptote** is a line which a function gets closer and closer to but never quite reache

HORIZONTAL ASYMPTOTES

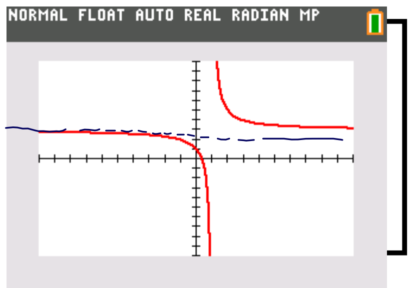
In **Chapter 18** we observed exponential functions such as $f(x) = 2^x + 0$.

We saw that as $x \to -\infty$, $f(x) \to 0$ from



For example, consider the function $g(x) = 3 + \frac{2}{x-1}$.

Sketch Curve



Using the trace feature, we observe:

- as $x \to -\infty$, $g(x) \to 3$ from below
- as $x \to \infty$, $g(x) \to 3$ from above.

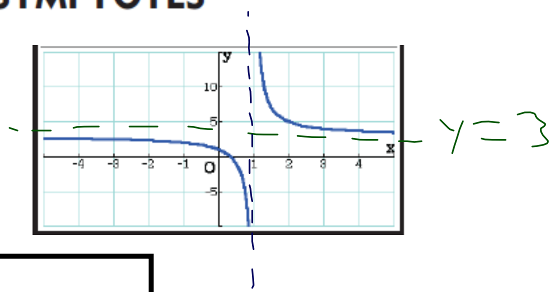
$y = 3$

We can check this by examining the function. As x gets very large, $\frac{2}{x-1}$ gets very small, so $g(x)$ approaches 3.

So, $g(x)$ has the horizontal asymptote $y = 3$.

VERTICAL ASYMPTOTES

The graph of $g(x) = 3 + \dfrac{2}{x-1}$



We observe there is a 'jump' or *discontinuity* in the graph when $x = 1$.

This occurs because $g(1) = 3 + \dfrac{2}{1-1} = 3 + \dfrac{2}{0}$ which is undefined.

As $x \to 1$ from the left, $g(x) \to -\infty$. As $x \to 1$ from the right, $g(x) \to \infty$.

Since $g(x)$ approaches, but never reaches, the line $x = 1$, we say that $x = 1$ is a **vertical asymptote** of the function.

For functions which contain a fraction, a vertical asymptote occurs when the denominator is zero.