C EXPONENTIAL EQUATIONS

An **exponential equation** is an equation in which the unknown occurs as part of the exponent or index.

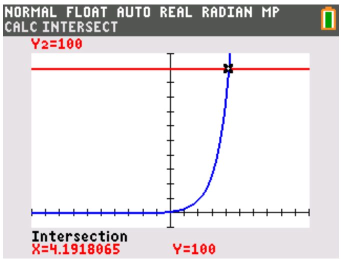
For example, $2^x = 50$ and $7^{1-x} = 40$ are both exponential equations.

We can use technology to solve exponential equations. We graph each side of the equation on the same set of axes, and the x-coordinate of the intersection point gives us the solution to the equation.

$y = 3^x$ $y = 100$

Use technology to solve the equation $3^x = 100$.

Tech.... $x = 4.19$



By Hand....

$\ln 3^x = \ln 100$

$\ln 3^x = \ln 100$

$\frac{x \ln 3}{\ln 3} = \frac{\ln 100}{\ln 3}$

without cal.

$x = \frac{\ln 100}{\ln 3}$

with $x = 4.19$

D GROWTH AND DECAY

$a > 1$ pos (x) $a < 1$ neg(x)

In this section we will examine situations where quantities are either increasing or decreasing exponentially. These situations are known as **growth** and **decay**, and occur frequently in the world around us.

For example, populations of animals, people, and bacteria usually *grow* in an exponential way. Radioactive substances, and items that depreciate in value, usually *decay* exponentially.

Exponential growth occurs for $y = a^x$ if $a > 1$.
Exponential decay occurs for $y = a^x$ if $0 < a < 1$.

An entomologist monitoring a grasshopper plague notices that the area affected by the grasshoppers is given by $A(n) = 1000 \times (1.15)^n$ hectares, where n is the number of weeks after the initial observation.

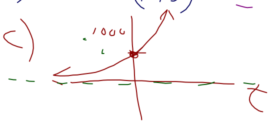
initial amount $a > 1$ Growth

a Find the original affected area. 1000 $t = 0$
b Find the affected area after: **i** 5 weeks **ii** 12 weeks.
c Draw the graph of $A(n)$ against n.
d How long will it take for the area affected to reach 10 000 hectares?

b) i) $1000 \times (1.15)^5 = 2011$

ii) $1000 \times 1.15^{12} = 5350$

c)



d) $10000 = 1000 \times (1.15)^n$

$\ln 10 = \ln 1.15^n$

$\ln 10 = \ln 1.15^n$ $\log 10 = \log 1.15^n$

$\ln 10 = n \ln 1.15$

$n = \frac{\ln 10}{\ln 1.15} = 16.47$ weeks

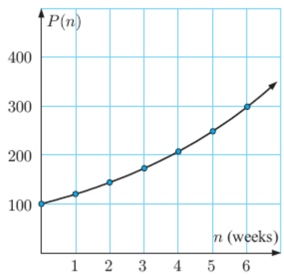
GROWTH

A population of 100 mice is increasing by 20% each week. To increase a quantity by 20%, we multiply it by 120% or 1.2.

If $P(n)$ is the population after n weeks, then

$P(0) = 100$ {the original population}
$P(1) = P(0) \times 1.2 = 100 \times 1.2$
$P(2) = P(1) \times 1.2 = 100 \times (1.2)^2$
$P(3) = P(2) \times 1.2 = 100 \times (1.2)^3$, and so on.

From this pattern we see that $P(n) = 100 \times (1.2)^n$, which is an exponential function.



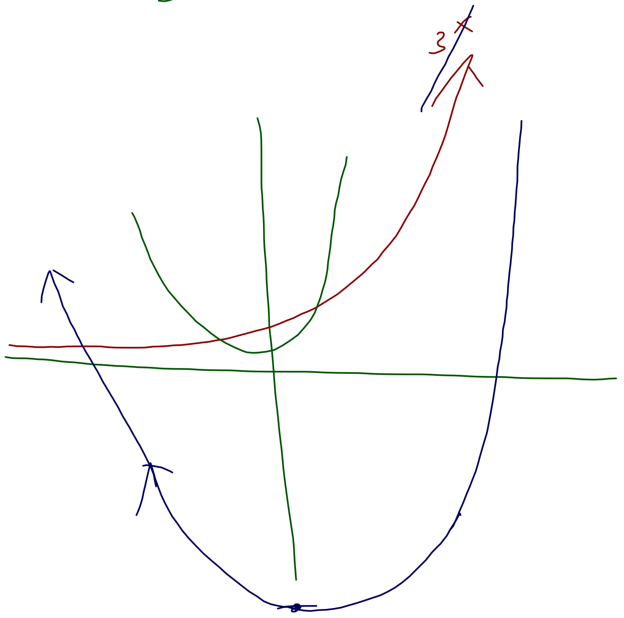
$$y = 100(.63)^x + 8$$

$a < 1$

Exp. Decay. by 37%

0	1	2
100	63	39.69

$$3^x = x^2 - 9$$



EXERCISE 18C

1 Solve using technology:

a $2^x = 20$ b $2^x = 100$

d $(1.2)^x = 3$ e $(1.04)^x = 4.238$

a) $2^x = 20$

$\ln 2^x = \ln 20$

$x \ln 2 = \ln 20$

$x = \frac{\ln 20}{\ln 2}$

2 Solve using technology:

a $3 \times 2^x = 93$ b $40 \times (0.8)^x = 10$

$\ln$ $2^x = 31$ $\ln$

$\ln 2^x = \ln 31$

$x \ln 2 = \ln 31$

$x = \frac{\ln 31}{\ln 2}$

EXERCISE 18D.1

1 A weed in a field covers an area of $A(t) = 3 \times (1.08)^t$ square metres after t days.

a Find the initial area the weed covered.

b Find the area after: i 2 days ii 10 days iii 30 days.

c Sketch the graph of $A(t)$ against t using the results of a and b only.

2 A breeding program to ensure the survival of pygmy possums was established with an initial population of 50. From a previous program, the expected population $P(n)$ in n years' time is given by $P(n) = 50 \times (1.23)^n$.

a What is the expected population after: i 2 years ii 5 years iii 10 years?

b Sketch the graph of $P(n)$ against n.

c How long will it take for the population to reach 600?