Chapter 16 Functions

A RELATIONS AND FUNCTIONS

(3,0) (4,0) (4,-2) (6,1)

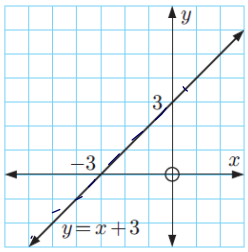
RELATIONS

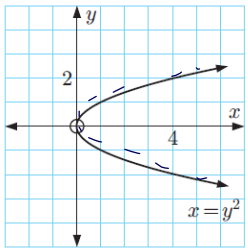
A **relation** is any set of points which connect two variables.

A relation is often expressed in the form of an **equation** connecting the **variables** x and y. In this case the relation is a set of points (x, y) in the **Cartesian plane**.

$y = x^2 + 8$ $y = 3x + 9$

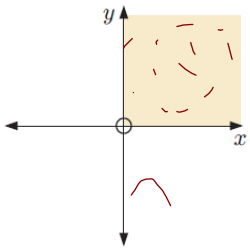
For example $y = x + 3$ and $x = y^2$ are the equations of two relations. Each equation generates a set of ordered pairs, which we can graph:





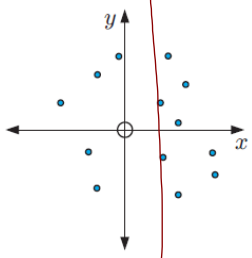
However, a relation may not be able to be defined by an equation. For example:

(1)



The set of all points in the first quadrant is a relation.
$x > 0, \quad y > 0$

(2)



These 13 points form a relation.

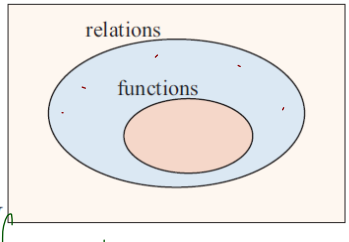
FUNCTIONS

A **function**, sometimes called a **mapping**, is a relation in which no two different ordered pairs have the same x-coordinate or first component.

We can see from the above definition that a function is a special type of relation.

Every function is a relation, but not every relation is a function.

This can be represented in set form as shown.



T or F

T or F

Every relation is a function

Every function is a relation

TESTING FOR FUNCTIONS

Algebraic Test:

If a relation is given as an equation, and the substitution of any value for x results in one and only one value of y, then the relation is a function.

For example:

$y = 3x - 1$ is a function, as for any value of x there is only one corresponding value of y

$x = y^2$ is not a function, since if $x = 4$ then $y = \pm 2$.

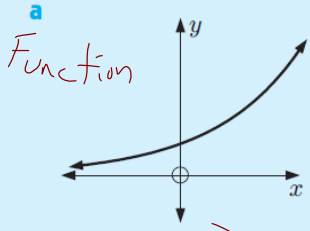
Geometric Test or Vertical Line Test:

If we draw all possible vertical lines on the graph of a relation, the relation:

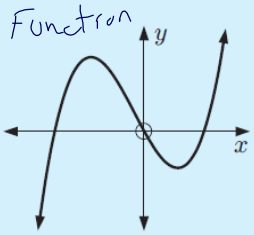
- is a function if each line cuts the graph no more than once
- is not a function if at least one line cuts the graph more than once.

Which of the following relations are functions?

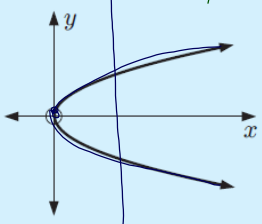
a



b

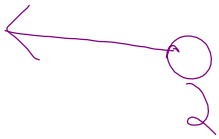


c



GRAPHICAL NOTE

- If a graph contains a small **open circle** such as —○, this point is **not included**.
- If a graph contains a small **filled-in circle** such as —●, this point **is included**.
- If a graph contains an **arrow head** at an end such as —→, then the graph continues indefinitely in that general direction, or the shape may repeat as it has done previously.

	Set Notation	Interval notation
	$x \geq 5$	$[5, \infty)$
	$2 < x \leq 7$	$(2, 7]$
	$-1 < x < 4$	$(-1, 4)$
	$x < 2$	$(-\infty, 2)$