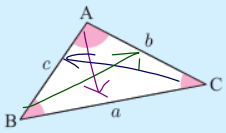
H THE SINE RULE

The **sine rule** is a set of equations which connects the lengths of the sides of any triangle with the sines of the angles of the triangle. The triangle does not have to be right angled for the sine rule to be used.

In any triangle ABC with sides a, b, and c units in length, and opposite angles A, B, and C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

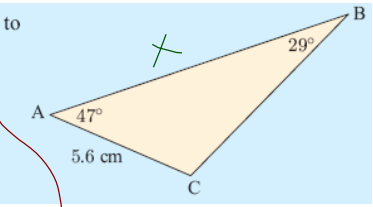


The sine rule is used to solve problems involving triangles, given:

- **two angles** and **one side** AAS or ASA
- **two sides** and a **non-included** angle. ASS SSA

FINDING SIDES (AAS or ASA)

Find the length of BC correct to three significant figures.



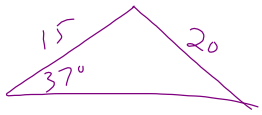
$\frac{\sin 29°}{5.6} = \frac{\sin 47}{a}$

$a \sin(29°) = 5.6 \sin(47)$

$a = \frac{5.6 \sin 47}{\sin 29}$

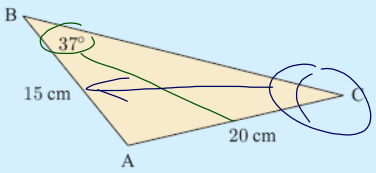
$a = 8.45$ cm

$\frac{\sin 104°}{x} = \frac{\sin 29}{5.6}$



FINDING ANGLES (SSA)

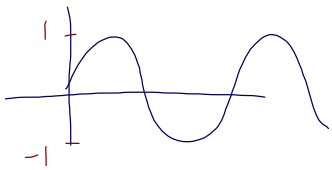
Determine the size of $\hat{ACB}$ correct to 3 significant figures.



$\frac{\sin 37}{20} = \frac{\sin C}{15}$

$15 \sin 37 = 20 \sin C$

$\frac{15 \sin 37}{20} = \sin C$



$C = \sin^{-1}\left(\frac{15 \sin 37}{20}\right)$

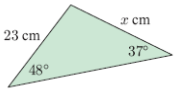
$C = \sin^{-1}(.451361274)$

$C = 26.8°$

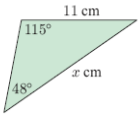
SAA

1 Find the value of x:

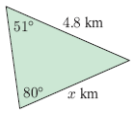
a



b



c



AAS

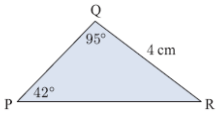
2 In triangle ABC, find:

a a if $A = 63°$, $B = 49°$, and $b = 18$ cm

b b if $A = 82°$, $C = 25°$, and $c = 34$ cm

3 Find the lengths of the remaining sides of triangle PQR.

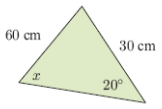
AAS



SSA

4) Find the value of x:

a



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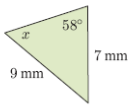
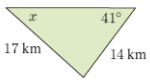


c



SSA

5) A triangle has vertices A, B, and C with opposite side lengths a, b, and c respectively. Find:

a $\hat{BAC}$ if $\hat{ABC} = 45°$, $a = 8$ cm, and $b = 11$ cm

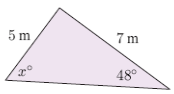
b $\hat{ABC}$ if $a = 32$ cm, $b = 23$ cm, and $\hat{BAC} = 42°$

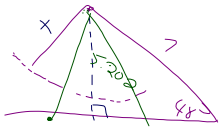
ASS

6) Unprepared for class, Mr Whiffen asks his students to determine the size of x in the diagram shown.

a Show that Mr Whiffen's question cannot be solved.

b Explain what this means about the triangle Mr Whiffen created.





a) $\frac{\sin x}{7} = \frac{\sin 48}{5}$

$7\sin 48 = 5\sin x$

$\frac{7\sin 48}{5} = \sin x$

$1.044\ldots = \sin x$

X does not exist.

Unknown lengths

$0 < x < 5.202$

No $\triangle$ can be formed

$x = 5.202$

one rt. $\triangle$ can be form

$5.202 < x < 7$

Two $\triangle$ can be formed

$x > 7$

only $\triangle$ can be formed