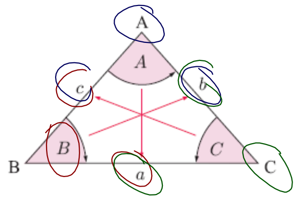
F AREAS OF TRIANGLES

$\frac{1}{2} \cdot b \cdot h$

If we know the base and height measurements of a triangle, we can calculate the area using **area** $= \frac{1}{2}$ **base** $\times$ **height**.

THE AREA OF A TRIANGLE FORMULA

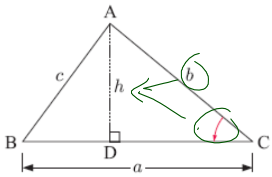
Suppose triangle ABC has angles of size A, B, and C, and the sides opposite these angles are labelled a, b, and c respectively.

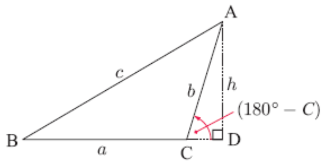


SAS = side/angle/side

included angle

Any triangle that is not right angled must be either acute or obtuse. In either case we construct a perpendicular from A to D on BC (extended if necessary).





Using right angled trigonometry:

$$\sin C = \frac{h}{b} \qquad \therefore \; h = b \sin C$$

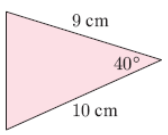
$$\sin(180° - C) = \frac{h}{b} \qquad \therefore \; h = b\sin(180° - C) \qquad \therefore \; h = b \sin C$$

So, area $= \frac{1}{2}ah$ gives $\boxed{A = \frac{1}{2}ab \sin C.}$ $A = \frac{1}{2} bc \sin A$ $A = \frac{1}{2} ac \sin B$

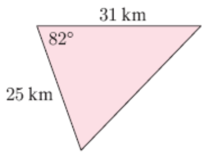
Using different altitudes we can show that the area is also $\frac{1}{2}bc\sin A$ or $\frac{1}{2}ac\sin B$.

1 Find the area of:

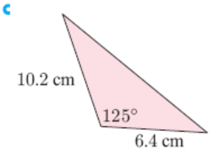
a



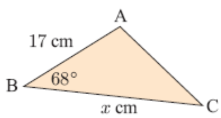
b



c



2 If triangle ABC has area 150 cm^2, find the value of x:



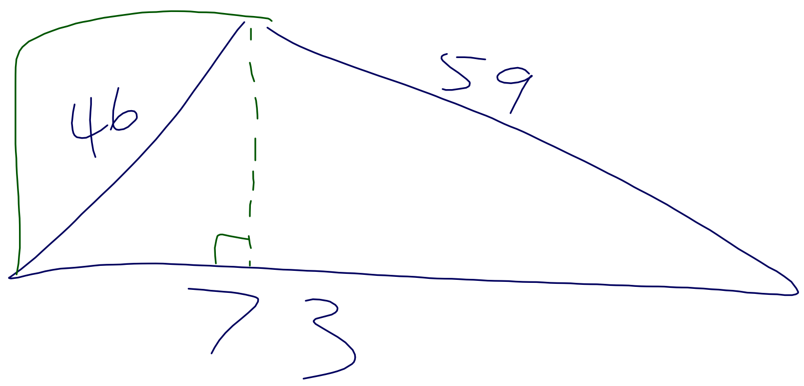
3 Calculate the area of:

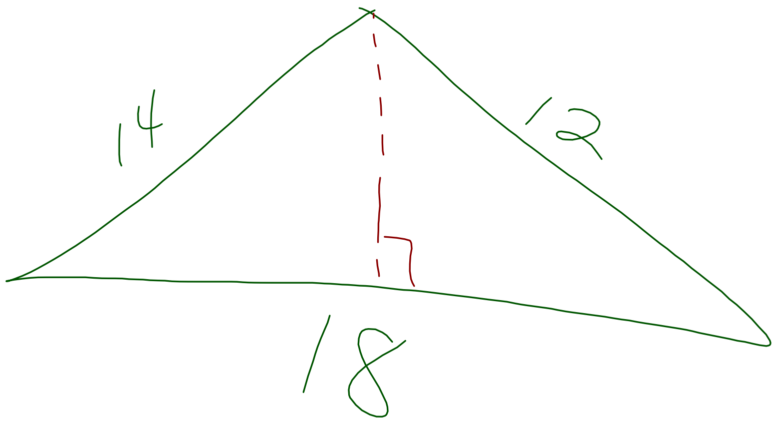
a an isosceles triangle with equal sides of length 21 cm and an included angle of 49°

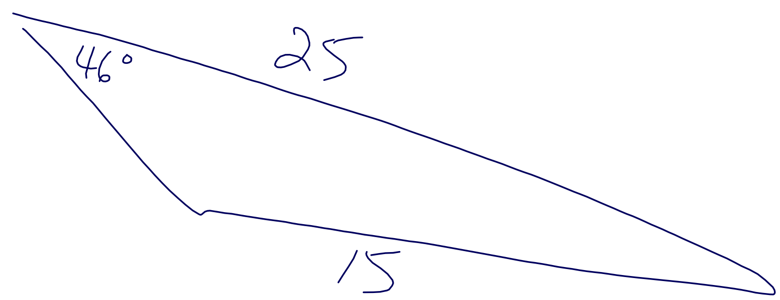
b an equilateral triangle with sides of length 57 cm.

4 A parallelogram has adjacent sides of length 4 cm and 6 cm. If the included angle measures 52°, find the area of the parallelogram.

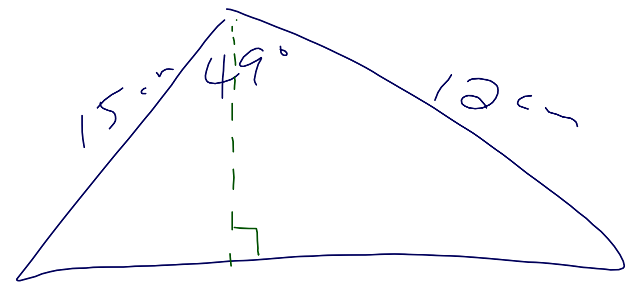
5 A rhombus has sides of length 12 cm and an angle of 72°. Find its area.


46
59
73


14
12
18

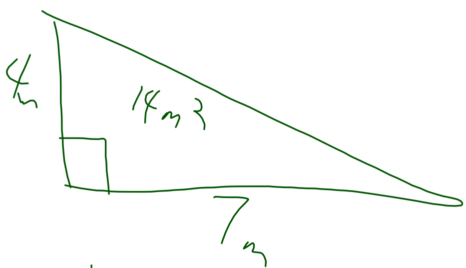


ASS SSA

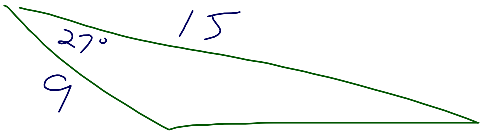


$$A = \frac{1}{2}(15)(12)\sin 49° \text{ cm}^2$$

$$= 54.3 \text{ cm}^2$$



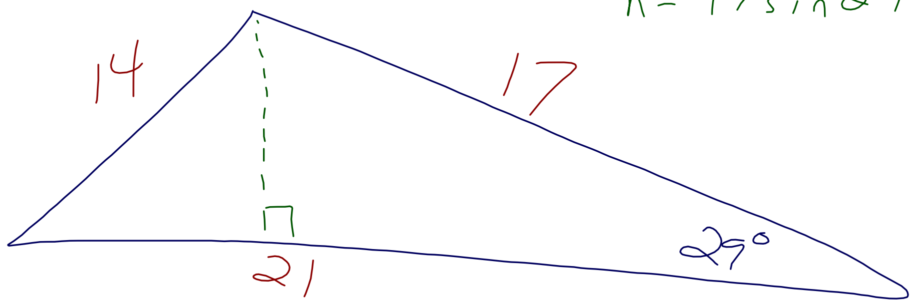
$$A = \frac{1}{2}(4)(7)\sin 90° = 14$$



$$A = \frac{1}{2}(9)(15)\sin 27^\circ$$

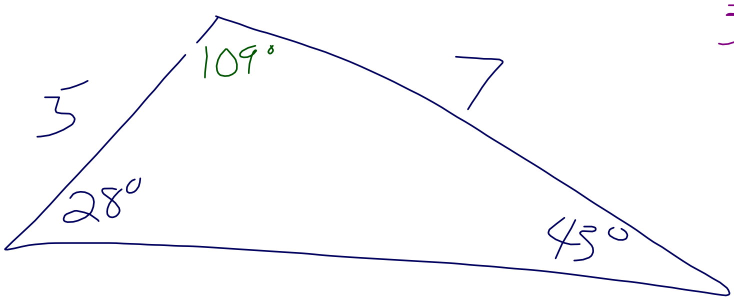
$\sin 29 = \frac{h}{17}$

$h = 17\sin 29$



$\frac{1}{2}(21)(17)\sin 29$

$180 - 28 - 43$



3

5

$A = \frac{1}{2}(5)(7)\sin 109°$

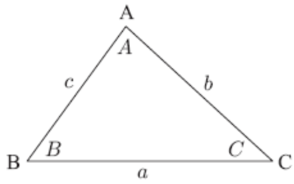

14
17
41

G THE COSINE RULE

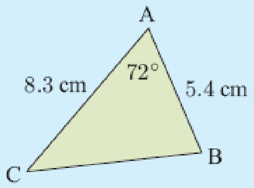
The **cosine rule** involves the sides and angles of any triangle. The triangle does not need to contain a right angle.

In any $\triangle$ABC with sides a, b, and c units in length, and opposite angles A, B, and C respectively:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{or} \quad b^2 = a^2 + c^2 - 2ac\cos B$$
$$\text{or} \quad c^2 = a^2 + b^2 - 2ab\cos C$$



Ex. Find the length BC:

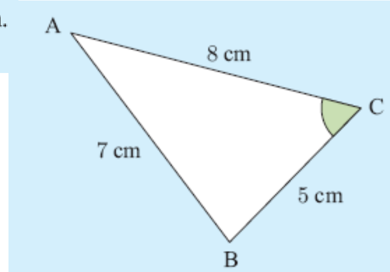


If we know all **three sides** of a triangle, we can rearrange the cosine rule formulae to find any of the angles:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

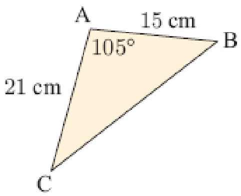
We use the **inverse** cosine ratio $\cos^{-1}$ to evaluate the angle.

Ex. In triangle ABC, AB = 7 cm, BC = 5 cm, and CA = 8 cm. Find the measure of angle BCA.

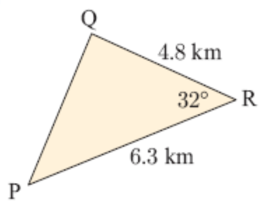


1 Find the length of the remaining side in the given triangle:

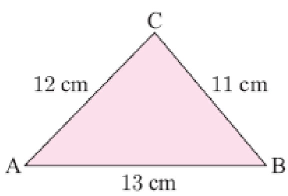
a



b

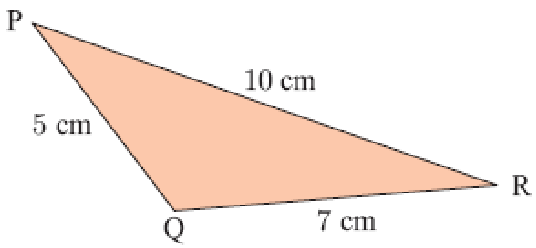


2 Find the measure of all angles of:

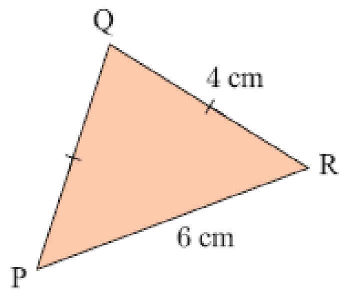


3 Find the measure of obtuse angle PQR:

a



b



4 **a** Find the smallest angle of a triangle with sides 11 cm, 13 cm, and 17 cm.

b Find the largest angle of a triangle with sides 4 cm, 7 cm, and 9 cm.