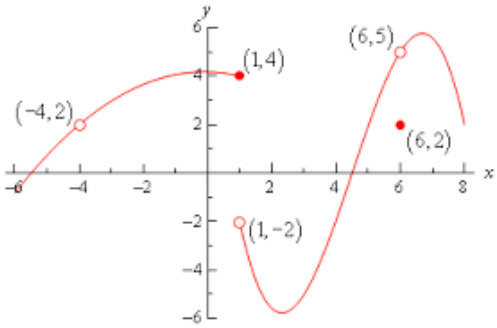
Review for Limits Quiz April 2017

2.1 Rates of change and limits.

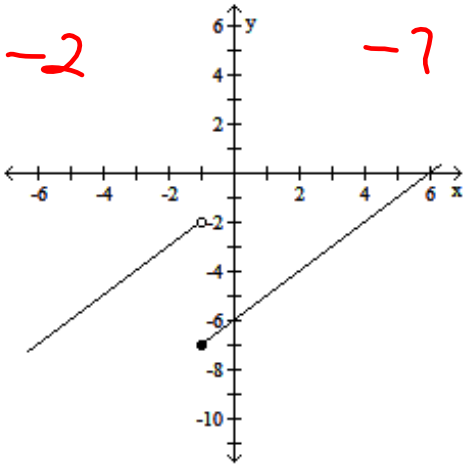
Understand that the limit as $x \to c$ never depends on how the function may or may not be defined at c.

In order for the limit to be defined as $x \to c$, the limit must be the same from the right and from the left.



Determine the limit graphically, if it exists.

5) Find $\lim_{x \to -1^-} f(x)$ and $\lim_{x \to -1^+} f(x)$.



−2 −7

lim DNE
x→−1

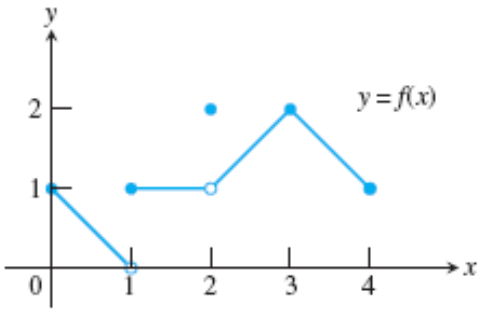


Figure 2.6 The graph of the function

$$f(x) = \begin{cases} -x + 1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ x - 1, & 2 < x \le 3 \\ -x + 5, & 3 < x \le 4. \end{cases}$$

At $x = 0$: $\lim_{x \to 0^+} f(x) =$ 1

At $x = 1$: $\lim_{x \to 1^-} f(x) =$ 0

$\lim_{x \to 1^+} f(x) =$ 1

At $x = 2$: $\lim_{x \to 2^-} f(x) =$ 1

$\lim_{x \to 2^+} f(x) =$ 1

$\lim_{x \to 2} f(x) =$ 1

At $x = 3$: $\lim_{x \to 3^-} f(x) =$ 2

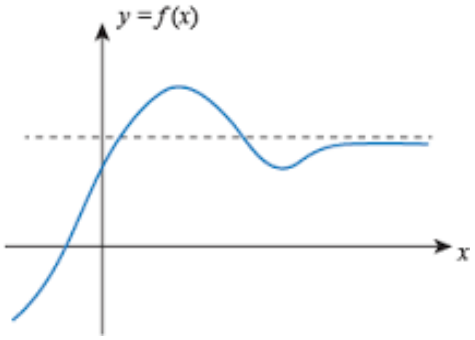
At $x = 4$: $\lim_{x \to 4^-} f(x) =$ 1

2.2 Limits involving infinity

A horizontal asymptote of the graph of a function is either:

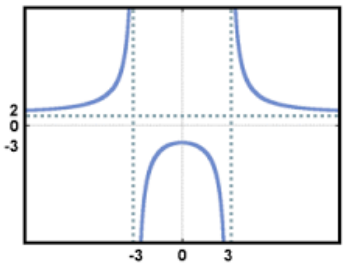
$\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

where b is a constant



The line x = a is a vertical asymptote of the graph of a function y = f(x) if either

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$



$\lim_{x \to 3^-} = -\infty$

$\lim_{x \to 3^+} = \infty$

Determine the limit by substitution.

1) $\lim_{x \to 2} (x^3 + 5x^2 - 7x + 1)$

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Determine the limit algebraically, if it exists.

2) $\lim_{x \to -4} \frac{x^2 - 16}{x + 4}$

$\lim_{x \to -4} \frac{2x}{1} = -8$

3) $\lim_{x \to 7} \frac{x^2 + 3x - 70}{x - 7}$

$\lim_{x \to 7} \frac{2x+3}{1}$

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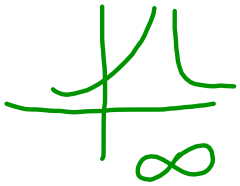
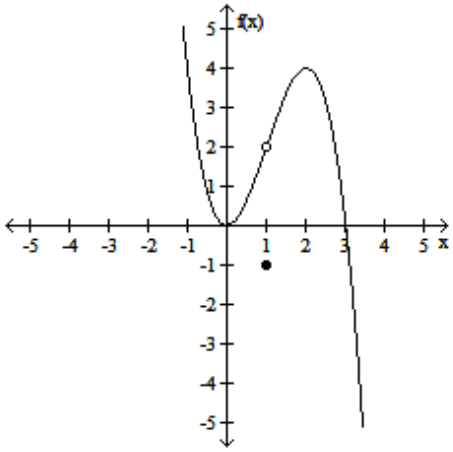
4) $\lim_{x \to 6} \frac{x + 6}{(x - 6)^2}$

DNE

$\frac{x+6}{(x-6)(x-6)}$



6) $\lim_{x \to 1} f(x)$



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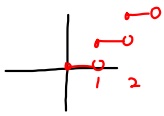
Find the indicated limit.

7) $\lim_{x \to -5^-} \text{int}\, x = -6$

$y = \lfloor x \rfloor$



$\lim_{x \to 1^-} = 0$

Find the limit.

8) $\lim_{x \to \infty} \frac{x^2 + 5x + 19}{x^3 + 3x^2 + 4}$

$\lim_{x \to 1^+} = 1$

0

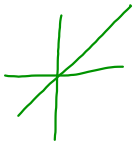
$\lim_{x \to \infty} \frac{7x^2 \ldots}{3x^2 \ldots} = \frac{7}{3}$

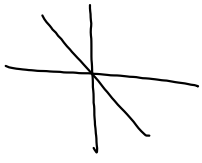
$\lim_{x \to \infty} \frac{7x^3}{3x^2} = \infty$

9) $\lim_{x \to -\infty} \frac{3x^3 + 4x^2}{x - 5x^2} = \infty$

end behavior $\frac{3x^3}{-5x^2} = -\frac{3}{5}x$

$\frac{7}{3}x$ ↓

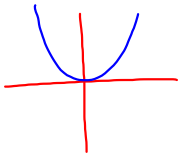




$\lim_{x \to -\infty} = \infty$

$\frac{7x^5 \ldots}{2x^3 \ldots}$ end behavior

$\frac{7}{2}x^2$

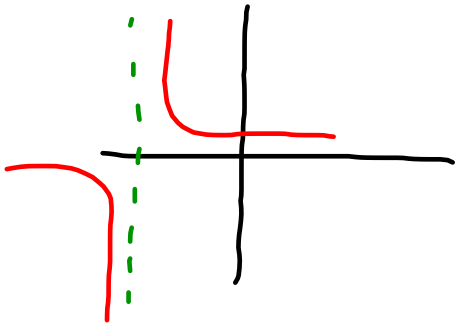


$\lim_{x \to \pm\infty} = \infty$

10) $\lim_{x \to (-2)^-} \frac{1}{x+2}$ $-\infty$

11) $\lim_{x \to (-2)^+} \frac{1}{x+2}$ ∞

12) $\lim_{x \to -2} \frac{1}{x+2}$ DNE



$y = \frac{1}{x}$

Find the vertical asymptotes of the graph of f(x).

13) $f(x) = \frac{x}{x+8}$

$x = -8$

horizontal

$y = 1$

Section 2.3 Continuity

<u>continuous functions:</u> can be traced without lifting your pencil

<u>points of discontinuity:</u> are points on the graph of a function where you need to lift your pencil

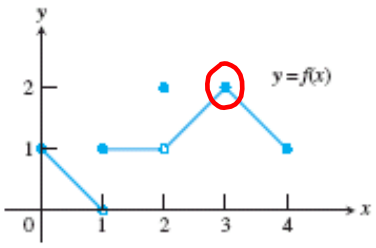


Figure 2.18 The function is continuous on [0, 4] except at $x = 1$ and $x = 2$. (Example 1)

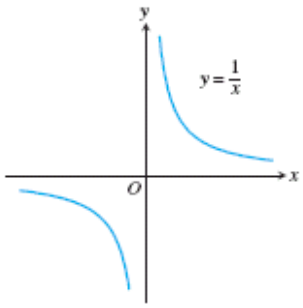


Figure 2.22 The function $y = 1/x$ is continuous at every value of x except $x = 0$. It has a point of discontinuity at $x = 0$. (Example 3)