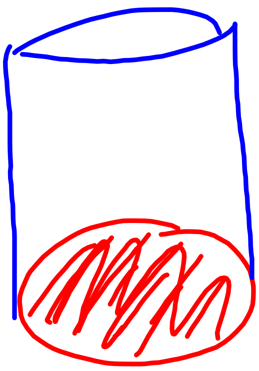
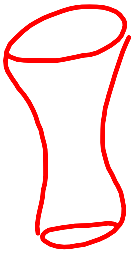


$V = \pi r^2 h$

$A \cdot h$



worksheet

Section 8.3 Day 1 #1, 15, 17, 19, 23-26

DEFINITION Volume of a Solid

The volume of a solid of known integrable cross section area $A(x)$ from $x = a$ to $x = b$ is the integral of A from a to b,

$$V = \int_a^b A(x)\, dx.$$

How to Find Volume by the Method of Slicing

1. Sketch the solid and a typical cross section.
2. Find a formula for $A(x)$.
3. Find the limits of integration.
4. Integrate $A(x)$ to find the volume.

Finding Volume

Find the area of a cross section & integrate

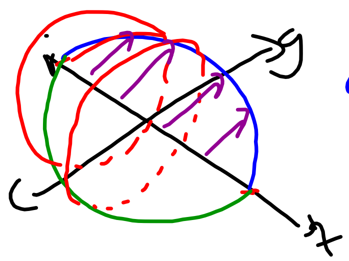
① Sketch a typical cross section

② Find a formula for the area

③ Find the limits of integration

④ Integrate A to find the volume

①



$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

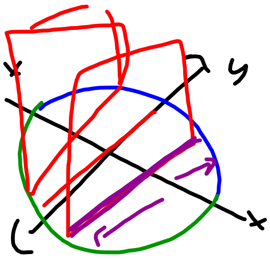
X-sections are circular disks with diameters in the x-y-plane

area $= \pi \cdot r^2$ radius = y value

Area $= \pi\left(\sqrt{1-x^2}\right)^2$ $= \sqrt{1-x^2}$

$\boxed{\text{Area} = \pi(1-x^2)}$ $V = \pi\int_{-1}^{1}(1-x^2)\,dx$

2



$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

Squares with bases in the xy plane

Area of a square S^2

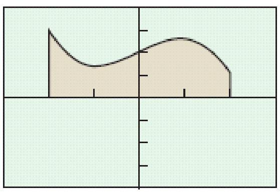
$S = 2\sqrt{1-x^2}$

Area $= (2\sqrt{1-x^2})^2$

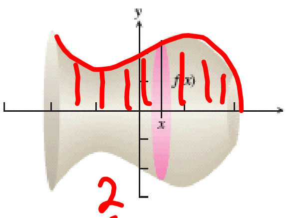
$4\int_{-1}^{1}(1-x^2)\,dx$ $\boxed{= 4(1-x^2)}$

EXAMPLE 2 A Solid of Revolution

The region between the graph of $f(x) = 2 + x\cos x$ and the x-axis over the interval $[-2, 2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.



[−3, 3] by [−4, 4]



$\pi\int_{-2}^{2}(2 + x\cos x)^2\,dx$

Cross section

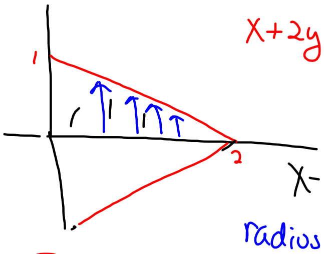
Calculator

#7

Volumes of Revolution

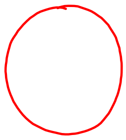
$x + 2y = 2 \rightarrow y = -\frac{1}{2}x + 1$

around x-axis



X-sections are disks

radius = y-value



Area of each disk $= \pi r^2$

$= \pi\left(-\frac{1}{2}x+1\right)^2$

$= \pi\left(\frac{1}{4}x^2 - x + 1\right)$

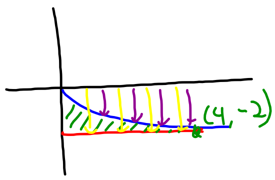
$V = \pi \int_0^2 \left(\frac{1}{4}x^2 - x + 1\right)dx$

$V = \pi\left[\frac{1}{12}x^3 - \frac{1}{2}x^2 + x\right]_0^2$

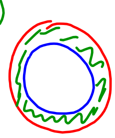
$V = \pi\left[\frac{8}{12} - 2 + 2\right]$ $V = \frac{8\pi}{12} = \boxed{\frac{2\pi}{3}}$

(20) Revolve around x-axis

$y = -\sqrt{x}$ $y = -2$ $x = 0$ washer



X-sections are washers



outside r

$\pi R^2 - \pi r^2$

inside rad

$R = 2$ $r = -\sqrt{x}$

$A = \pi 2^2 - \pi(-\sqrt{x})^2$

$A = 4\pi - x\pi$

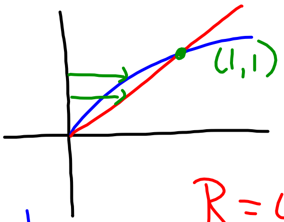
$V = \pi \int_0^4 (4 - x)dx$

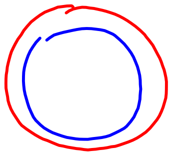
$V = \pi\left[4x - \frac{1}{2}x^2\right]_0^4$

$= \pi[16 - 8] = 8\pi$

(28) $x = y^2 \quad x = y$

$y = \sqrt{x} \quad y = x$ around y-axis





$R = y \quad r = y^2$

$$\pi \int_0^1 [y^2 - (y^2)^2]\,dy$$

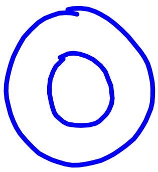
$$\pi \left[\frac{1}{3}y^3 - \frac{1}{5}y^5\right]_0^1$$

$$\pi \left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\pi \left[\frac{5}{15} - \frac{3}{15}\right] = \boxed{\frac{2\pi}{15}}$$

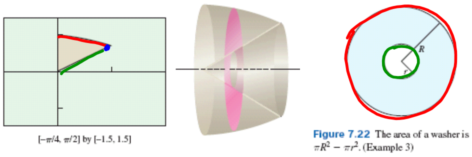
Disc OR

Washer



EXAMPLE 3 Washer Cross Sections

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.



[−π/4, π/2] by [−1.5, 1.5]

Figure 7.22 The area of a washer is $\pi R^2 - \pi r^2$. (Example 3)

$$A = \pi R^2 - \pi r^2$$

$$V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\,dx$$

$$V = \pi \int_0^{\frac{\pi}{4}} (\cos 2x)\,dx$$

$$\pi \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$

$$\frac{1}{2}\pi \left[\sin \frac{\pi}{2} - \sin 0\right]$$

$$\frac{1}{2}\pi (1-0) \quad \boxed{\frac{\pi}{2}}$$