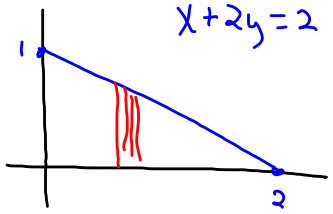
8.3 Volumes from a revolution

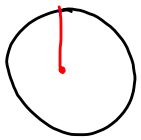
(7)



radius = y-value

$2y = -x + 2$

$r = y = -\frac{1}{2}x + 1$



$A = \pi r^2$

$A = \pi\left(\frac{1}{4}x^2 - x + 1\right)$

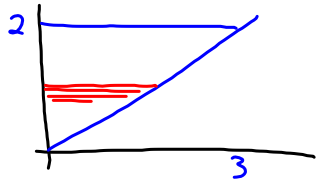
$\pi \int_0^2 \left(\frac{1}{4}x^2 - x + 1\right) dx$

$\pi\left[\frac{1}{12}x^3 - \frac{1}{2}x^2 + x\right]_0^2$

$\pi\left[\frac{8}{12} - 2 + 2\right] - 0$

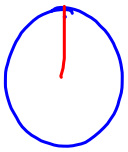
$\pi\left(\frac{8}{12}\right)$ $\boxed{\frac{2\pi}{3}}$

(8)



$x = \frac{3}{2}y$

radius = x-value



$\pi r^2 \quad r = \frac{3}{2}y$

$Area = \pi\left(\frac{3}{2}y\right)^2$

$\pi\left(\frac{9}{4}y^2\right)$

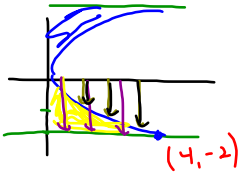
$\frac{9\pi}{4}\int_0^2 y^2\,dy$

$\frac{9\pi}{4}\left[\frac{1}{3}y^3\right]_0^2$

$\frac{9\pi}{4}\cdot\frac{8}{3} = \frac{72\pi}{12} = \boxed{6\pi}$

Washers

(20) $y = -\sqrt{x} \quad y = -2 \quad x = 0 \quad$ x-axis

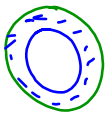


Outside Radius

$R = 2 \quad R^2 = 4$

inside radius

$r = \sqrt{y} \quad r^2 = x$



$A = \pi R^2 - \pi r^2$

$A = \pi\left(R^2 - r^2\right)$

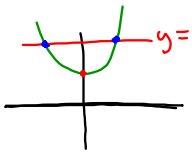
$A = \pi(4 - x)$

$V = \pi\int_0^4 (4 - x)\,dx$

$\pi\left[4x - \frac{1}{2}x^2\right]_0^4$

$\pi(16 - 8) = \boxed{8\pi}$

(19) $y = \sec x \quad y = \sqrt{2} \quad [-\frac{\pi}{4}, \frac{\pi}{4}]$



$\sec x = \sqrt{2}$

$\cos x = \frac{1}{\sqrt{2}}$

$R = \sqrt{2} \quad R^2 = 2$

Area $r = \sec x \quad r^2 = \sec^2 x$

$\pi(2 - \sec^2 x)$

$\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (2 - \sec^2 x)\,dx \qquad 2\pi \int_{0}^{\frac{\pi}{4}} (2 - \sec^2 x)\,dx$

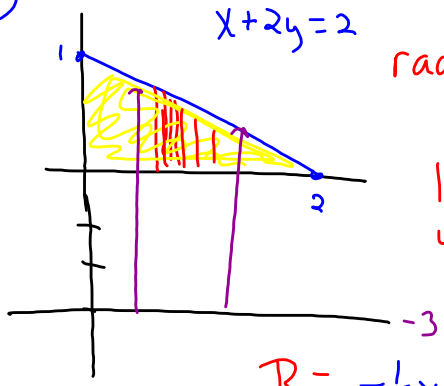
$2\pi [2x - \tan x]_0^{\frac{\pi}{4}}$

$2\pi [\frac{\pi}{2} - 1 - [0 - 0]]$

$2\pi [\frac{\pi}{2} - 1]$

$\pi^2 - 2\pi$

(7)



radius = y-value

$2y = -x + 2$

$y = -\frac{1}{2}x + 1$

$y = -3$

$R = -\frac{1}{2}x + 1 + 3$

$-\frac{1}{2}x + 4$

$\pi(-\frac{1}{2}x + 4)^2 - \pi(3)^2$

$r = 3$

$\pi(\frac{8}{12}) \quad \frac{2\pi}{3}$