8.2 Areas in a plane
HW: 3,6,9,16,18,21

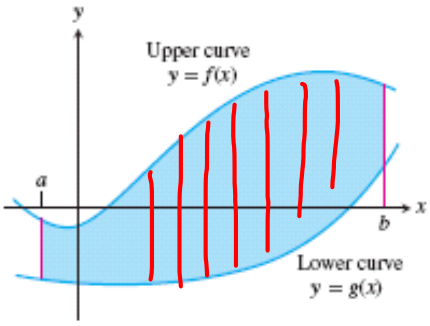


Figure 7.3 The region between $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$.

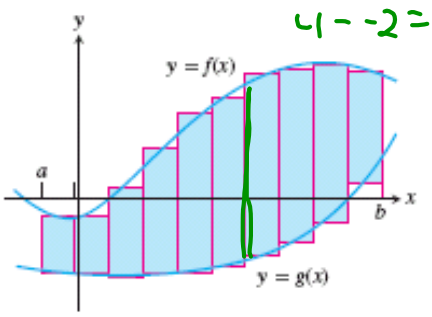
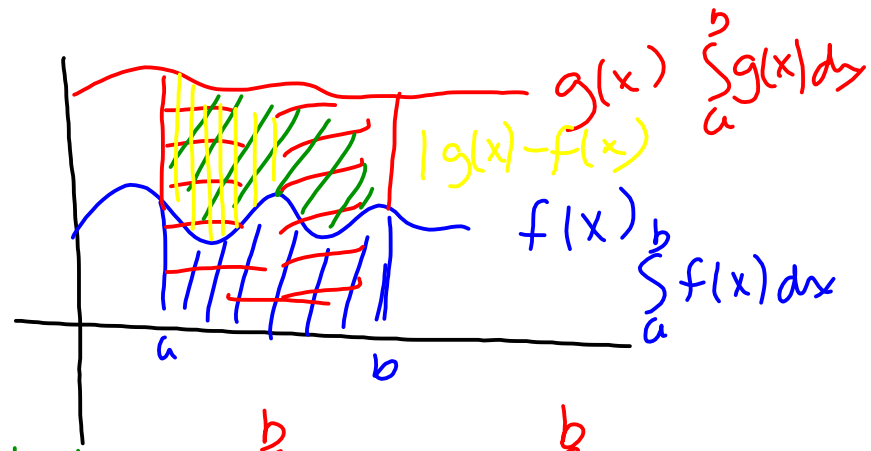


Figure 7.4 We approximate the region with rectangles perpendicular to the x-axis.

$$\int_a^b [f(x) - g(x)]\,dx.$$



$\int_a^b g(x)\,dx$

$|g(x) - f(x)|$

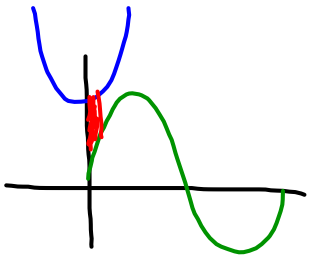
$\int_a^b f(x)\,dx$

between $\int_a^b g(x)\,dx - \int_a^b f(x)\,dx$

$$\int_a^b [g(x) - f(x)]\,dx$$

Area between curves:

Example 1: Find the area of the region between $y=\sec^2 x$ and $y=\sin x$ from $x=0$ to $x=\pi/4$.



$$\int_0^{\frac{\pi}{4}} [\sec^2 x - \sin x]\,dx$$

$$\tan x + \cos x \Big|_0^{\frac{\pi}{4}}$$

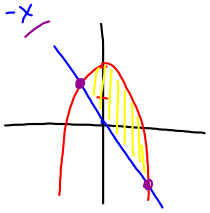
$$\tan\tfrac{\pi}{4} + \cos\tfrac{\pi}{4} - [\tan 0 + \cos 0]$$

$$1 + \frac{\sqrt{2}}{2} - 0 - 1 = \frac{\sqrt{2}}{2}$$

Area enclosed by intersecting curves:

Example 2: Find the area of the region enclosed by $y=2-x^2$ and the line $y=-x$.

$y=-x$ $\quad y=2-x^2$ $\quad y=-x^2+2$



$-x = 2-x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1)=0$

$x=2 \quad x=-1$

$$\int_{-1}^{2} [2-x^2-(-x)]\,dx$$

$$\int_{-1}^{2} [-x^2+x+2]\,dx$$

$$-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_{-1}^{2}$$

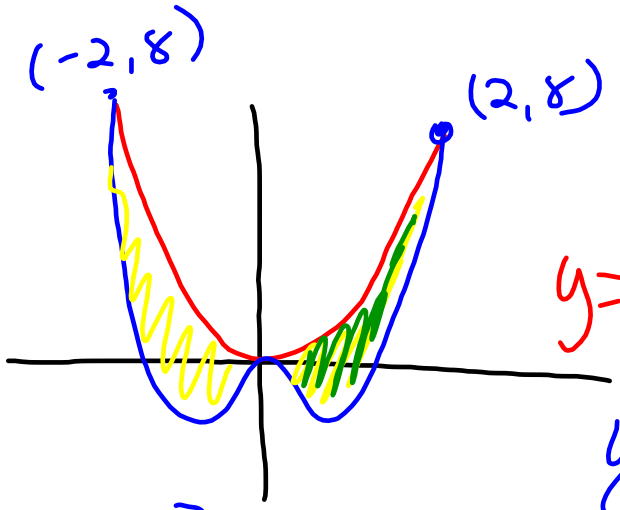
$$-\frac{8}{3} + 2 + 4 - \left[\frac{1}{3} + \frac{1}{2} - 2\right]$$

$$\left(-\frac{8}{3}\right) + 6 \left(-\frac{1}{3}\right) - \frac{1}{2} + 2$$

$$-3 + 8 - \frac{1}{2}$$

$5 - \frac{1}{2} = (4.5) \left(\frac{9}{2}\right)$

#5



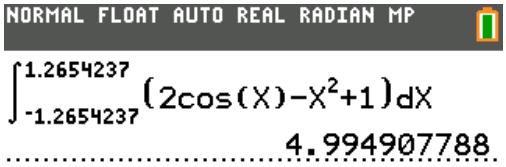
$y=2x^2$

$y=x^4-2x^2$

$\int_{-2}^{2}[f(x)-g(x)]dx$ $\quad 2\left[\int_{0}^{2}[f(x)-g(x)]dx\right]$

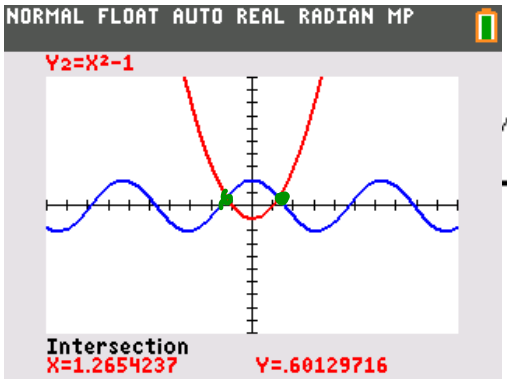
Using a calculator:

Example 3: Find the area of the region enclosed by the graphs of $y=2\cos x$ and $y=x^2-1$.



3 decimal places

4.995 or 4.994



Boundaries with changing functions:

Example 4: Find the area of region R in the first quadrant that is bounded above by $y=\sqrt{x}$ and below by the x axis and the line $y=x-2$.