Definite integral

$\int_a^b$ number

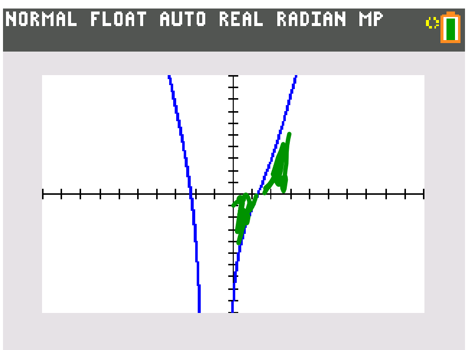
Section 8.1 HW: 1-11, odd, 12-17, 20-22

Integrals as a net change

Example 1: Interpreting a velocity function

$\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2}$ cm/sec $0 \leq t \leq 5$

describe the particle's movement

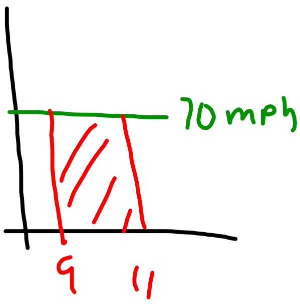


$v(t) = t^2 - \frac{8}{(t+1)^2}$

left $(0, 1.25)$ $(1.25, 5)$ right

When you want to find the cumulative affect of a varying rate of change

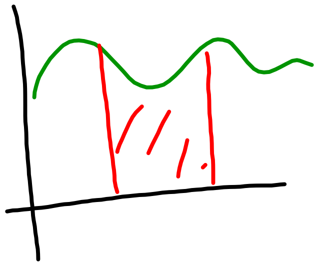
integrate it



$$\int_9^{11} 70\,dt$$

$$70t\Big|_9^{11}$$

$$70(11-9)=140$$



Example 2: Finding position from dispacement (using example 1)

displacement=change in position if + then it moves to the right, if - then it moves to the left

during the first second

Position at 1 second

Position at 5 seconds

Final position = initial position + displacement

To find displacement - Integrate velocity

Displacement after 1 sec

$s(0) = 9$

$v(t) = t^2 - \frac{8}{(t+1)^2}$

$\int_0^1 \left[t^2 - 8(t+1)^{-2}\right] dt$

$\frac{1}{3}t^3 + 8(t+1)^{-1} \Big|_0^1$

$\frac{1}{3}t^3 + \frac{8}{t+1} \Big|_0^1$

$\frac{1}{3} + 4 - (8)$

$\frac{13}{3} - \frac{24}{3} = -\frac{11}{3}$

displacement : $-\frac{11}{3}$

position $9 - \frac{11}{3}$

$\frac{27}{3} - \frac{11}{3} = \boxed{\frac{16}{3}}$

Integrating Velocity gives displacement

(net area between the curve and x-axis)

Integrating |velocity| = distance traveled

Example 4: A car moving with an initial velocity of 5 mph accelerates at the rate of a(t)=2.4t mph per second for 8 seconds.

a. How fast is the car going @ 8 sec?

$$5+\int_0^8 2.4t\,dt$$

$$5+\left[1.2t^2\right)_0^8$$

$\cancel{6.4}$

$$\begin{array}{r} 64 \\ 1.2 \\ \hline 128 \\ 64 \\ \hline 76.8 \end{array}$$

$76.8+5=81.8$

b. How far did the car travel?

*The integral isn't only useful in calculating distance and velocity. It is also useful in calculating growth, decay and consumption. To find the cummulative effect of a varying rate of change, integrate it.

Example: Potato consumption from 1970-1980
The rate of consumption=c(t)=2.2+1.1^t millions of bushels per year. How many bushels were consumed from the beginning of 1972 to the end of 1973?

$$c(t)=2.2+1.1^t$$

$$\int_2^4 2.2+1.1^t\,dt$$

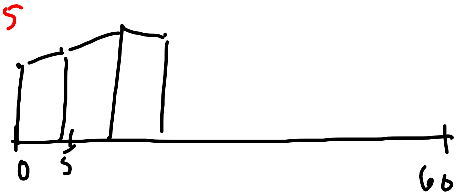
Example 6: Net change from data
No equation is accessible, but a chart of values with varying rates is given. Since the time intervals are space evenly, we car use the trapezoid rule to calculate the number of gallons.
How many gallons were pumped during the given hour?

time(min)	rate(gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

59, 62.5, 64.5, 61, 57.5, 56, 55, 57, 59.5, 60, 61.5, 63

$5(59+62.5+64.5+61 \cdots$

$\cdots 63)$

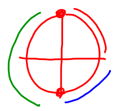


8.1 #1

$v(t)=5\cos t \qquad 0\le t\le 2\pi$

a) $(0,\frac{\pi}{2})$ right $(\frac{3\pi}{2},2\pi)$ right

$(\frac{\pi}{2},\frac{3\pi}{2})$ left



Stopped $t=\frac{\pi}{2}$ $t=\frac{3\pi}{2}$

b) $\int_0^{2\pi} 5\cos t\,dt$

$5\sin t\Big|_0^{2\pi}$

$5[\sin 2\pi-\sin 0]$

$5(0-0)$

→ displacement $=0$

$s(0)=3 \quad 3+0=3$ position

c) total distance traveled

$\int_0^{\frac{\pi}{2}} v(t)\,dt + \left|\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} v(t)\,dt\right| + \int_{\frac{3\pi}{2}}^{2\pi} v(t)\,dt$

$5[\sin\frac{\pi}{2}-\sin 0]$ $\qquad 5[\sin\frac{3\pi}{2}-\sin\frac{\pi}{2}]$ $\qquad 5[\sin 2\pi-\sin\frac{3\pi}{2}]$

$5(1-0)$ $\qquad 5(-1-1)$ $\qquad 5(0--1)$

5 $\qquad 5(-2)=-10$ $\qquad 5$

10

total distance traveled $=20$

$v(t) = 5\cos t$