7.4 Day 1 Exponential Growth and Decay
HW 1-9 odd

Separable Differential Equations

$\frac{dy}{dx} = f(y)g(x)$

*we separate it so the x's are on one side and the y's on the other
*then antidifferentiate each side with respect to its variable
*only one +C is needed

Example: Solve for y $\frac{dy}{dx} = (xy)^2$ when y=1 and x=1

$\frac{dy}{dx} = (xy)^2 \qquad (1,1)$

$\frac{dy}{y^2}$ $\qquad \frac{dy}{dx} = x^2y^2$

$y^{-2}dy = x^2dx$

$-y^{-1} = \frac{1}{3}x^3 + C \qquad (1,1)$

$-\frac{1}{y} = \frac{1}{3}x^3 + C$

$-1 = \frac{1}{3} + C$

$C = -\frac{4}{3}$

$-\frac{1}{y} = \frac{1}{3}x^3 - \frac{4}{3}$

$-\frac{1}{y} = \frac{x^3-4}{3}$

$-y = \frac{3}{x^3-4}$

$\boxed{y = \frac{3}{4-x^3}} \qquad y = -\frac{3}{x^3-4}$

2. $\frac{dy}{dx} = \frac{-x}{y}$ when y=3 and x=4

$$\frac{dy}{dx} = -\frac{x}{y} \quad (4,3)$$

$$y\,dy = -x\,dx$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C \quad (4,3)$$

$$\frac{1}{2}(3)^2 = -\frac{1}{2}(4^2) + C$$

$$\frac{9}{2} = -\frac{16}{2} + C$$

$$\frac{25}{2} = C$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \frac{25}{2}$$

$$y^2 = 25 - x^2$$

$$y = \sqrt{25-x^2} \quad (4,3)$$

8. $\frac{dy}{dx} = e^{x-y}$ when $y=2$ and $x=0$

$$\frac{dy}{dx} = e^{x-y} \qquad (0,2)$$

$$\frac{dy}{dx} = e^{x} \cdot e^{-y} \qquad \frac{dy}{e^{-y}}$$

$$e^{y}\,dy = e^{x}\,dx \qquad (0,2)$$

$$e^{y} = e^{x} + C$$

$$e^{2} = e^{0} + C$$

$$e^{2} = 1 + C$$

$$C = e^{2} - 1$$

$$e^{y} = e^{x} + e^{2} - 1 \quad \text{(ln)}$$

(ln)

$$y = \ln(e^{x} + e^{2} - 1)$$

$\frac{dy}{dx} = \cos^2 y \qquad (0,0)$

$\frac{dy}{\cos^2 y} = 1\,dx$

$\sec^2 y\,dy = 1\,dx$

$\tan y = x + C \qquad (0,0)$

$\tan 0 = 0 + C \qquad C = 0$

$\tan y = x$

$y = \tan^{-1} x$

$$\frac{dy}{dt} = ky$$

k is a constant
rate of change is directly proportional to amount present

$$\frac{1}{y}\,dy = k\,dt$$

$$\ln y = kt + C$$

$$y = e^{kt+C}$$

$$y = e^{C} \cdot e^{kt}$$

$$y = y_0 e^{kt}$$

$$y = y_0 e^{kt}$$

$$A = Pe^{rt}$$

$$y = Ae^{kt}$$

exponential growth

$k > 0$ $k < 0$ decay

compound interest

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

r = rate

n = # times Comp

↳ $A = Pe^{rt}$

How long to double $

7% compounded continuously?

$A = Pe^{rt}$

$2{,}000{,}000 = 1{,}000{,}000\, e^{.07t}$

$2 = e^{.07t}$

$\frac{A}{P} = 2$

$\frac{\ln 2}{.07} = \frac{.07t}{.07}$

Time Double

$\frac{\ln 2}{\text{rate}}$ $\frac{\ln 2}{\text{time}}$

$t = 9.9$ years

Carbon-14 dating

half life

5700 years

300 g : How much is left after 10,000 years?

$A = 300\left(\frac{1}{2}\right)^{\frac{10,000}{5,700}}$

$A = 88.920$ grams

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

(26) $y = y_0 e^{-.0005t}$

decay what is the half-life?

$\frac{y}{y_0} = \frac{1}{2}$

$\frac{1}{2} = e^{-.0005t}$

$\frac{\ln \frac{1}{2}}{-.0005} = \frac{-.0005t}{-.0005}$

$t = 1386$ years