4.4 Day 2

Derivative of ln x

$y = \ln x$

$e^y = x$

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$\boxed{\frac{dy}{dx} = \frac{1}{x}}$

---

(55) $f(x) = 2^x$

$f'(x) = 2^x \ln 2$

a) $f'(0) = 2^0 \ln 2$

$f'(0) = \ln 2$

b)

$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$\lim_{h \to 0} \frac{f(t) - f(0)}{t}$

$f'(0)$

$\lim_{h \to 0} \frac{2^h - 1}{h}$

$\ln 2$

$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$\cos x$

$\lim_{h \to 0} \frac{\sin(\frac{\pi}{2} + h) - \sin\frac{\pi}{2}}{h}$

$\cos x$

$\cos(\frac{\pi}{2}) = 0$

$\frac{d}{dx}\ln x = \frac{1}{x}$ $\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$

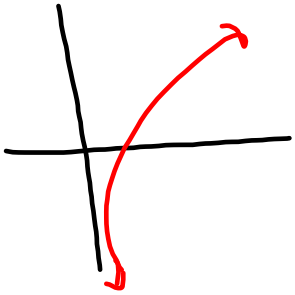
$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$ $\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}$

Domain: Derivatives are only defined where the function is defined:

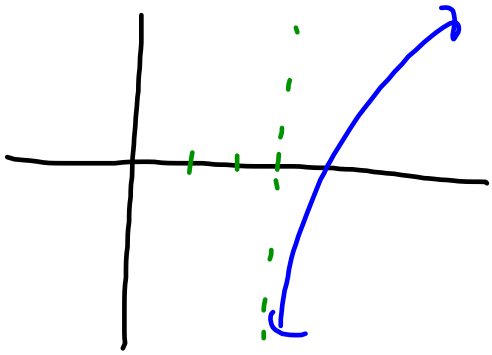
$x > 3$

$y = \ln x$ $x > 0$

Domain $(0, \infty)$



$y = \ln (x-3)$ $x - 3 > 0$



Ex 1 Find dy/dx

$$y = \ln(x^2 + 1)$$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$\frac{dy}{dx} = \frac{1}{x^2+1} \cdot 2x$

$$\boxed{\frac{dy}{dx} = \frac{2x}{x^2+1}}$$

EXAMPLE 3 A Tangent Through the Origin

A line with slope m passes through the origin and is tangent to the graph of $y = \ln x$. What is the value of m?

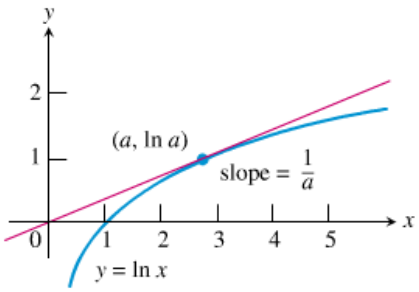


Figure 4.17 The tangent line intersects the curve at some point $(a, \ln a)$, where the slope of the curve is $1/a$. (Example 3)

So, if u is a differentiable function of x and $u > 0$, the formula is as follows.

For $a > 0$ and $a \neq 1$,

$$\frac{d}{dx}\log_a u = \frac{1}{u \ln a}\frac{du}{dx}.$$

(16) $y = (\ln x)^2$

$\frac{dy}{dx} = 2(\ln x)^1 \cdot \frac{1}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x}$

(18) $y = \ln\left(\frac{10}{x}\right)$

$u = 10x^{-1}$

$\frac{du}{dx} = -10x^{-2}$

$= \frac{-10}{x^2}$

$\frac{dy}{dx} = \frac{1}{\frac{10}{x}} \cdot \frac{-10}{x^2}$

$= \frac{x}{10} \cdot \frac{-10}{x^2}$

$\frac{dy}{dx} = \frac{-1}{x}$

(20) $y = x \cdot \ln x - x$

$x \cdot \frac{1}{x} + 1 \cdot \ln x - 1$

$1 + \ln x - 1$

$= \ln x$

(23) $y = \log_5 \sqrt{x}$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}\ln 5} \cdot \frac{1}{2\sqrt{x}}$

$u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\frac{1}{2\sqrt{x}}$

$$\boxed{\frac{dy}{dx} = \frac{1}{2x\ln 5}}$$

(26) $y = \log_3(1 + x\ln 3)$

$u = 1 + x\ln 3$

$u' = \ln 3$

$\frac{1}{(1+x\ln 3)\ln 3} \cdot \ln 3$

$x \cdot \ln 3$

$x \cdot 0 + 1 \cdot \ln 3$

$$\frac{dy}{dx} \quad \boxed{\frac{1}{1+x\ln 3}}$$