4.3 Derivatives of Inverse Trig Functions

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}, x\neq\pm1$$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}, x\neq\pm1$$

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}, x\neq\pm1,0$$

$$\frac{d}{dx}\left(\csc^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}, x\neq\pm1,0$$

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EXAMPLE 1 Applying the Formula

$$\frac{d}{dx}(\sin^{-1} \underline{x^2})$$

u

$$\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$$

$$\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$\frac{2x}{\sqrt{1-x^4}}$$

The derivative is defined for all real numbers. If u is a differentiable function of x, we get the Chain Rule form:

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}.$$

EXAMPLE 2 A Moving Particle

A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$. What is the velocity of the particle when $t = 16$?

$$\frac{1}{1+u^2}\frac{du}{dx} \qquad u=\sqrt{t} \qquad t^{\frac{1}{2}}$$

$$\frac{du}{dt} = \frac{1}{2}t^{-\frac{1}{2}} \boxed{= \frac{1}{2\sqrt{t}}}$$

$$\frac{1}{1+(\sqrt{t})^2}\cdot\frac{1}{2\sqrt{t}}$$

$$\frac{1}{1+t}\cdot\frac{1}{2\sqrt{t}} \qquad t=16$$

$$\frac{1}{1+16}\cdot\frac{1}{2\sqrt{16}}$$

$$\frac{1}{17}\cdot\frac{1}{8} = \frac{1}{136}$$

EXAMPLE 3 Using the Formula

$$\frac{d}{dx}\sec^{-1}(5x^4)$$

$$\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{1}{|5x^4|\sqrt{(5x^4)^2-1}}\cdot 20x^3$$

$$\frac{20x^3}{5x^4\sqrt{25x^8-1}}$$

$$\boxed{\frac{4}{x\sqrt{25x^8-1}}}$$

⑤ $y = \sin^{-1}\left(\frac{3}{t^2}\right)$

$3t^{-2}$

$-6t^{-3}$

$\frac{-6}{t^3}$

$\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}} \cdot \frac{-6}{t^3}$

$\frac{-6}{t^3\sqrt{1-\frac{9}{t^4}}}$

$\frac{-6}{t^3\sqrt{\frac{t^4}{t^4}-\frac{9}{t^4}}}$

$\frac{-6}{t^3\sqrt{\frac{t^4-9}{t^4}}}$

$\frac{-6}{t^3\sqrt{\frac{1}{t^4}(t^4-9)}}$

$\frac{-6}{t^3\frac{1}{t^2}\sqrt{t^4-9}}$

$\frac{-6}{t\sqrt{t^4-9}}$

$\frac{-6}{t^3\sqrt{1-\frac{9}{t^4}}}$

$\frac{-6}{t\cdot t^2\sqrt{1-\frac{9}{t^4}}}$

$\frac{-6}{t\sqrt{t^4}\sqrt{1-\frac{9}{t^4}}}$

$\boxed{\frac{-6}{t\sqrt{t^4-9}}}$