Complex Zeros and the Fundamental Theorem of Algebra

- Two Major Theorems
- Complex Conjugate Zeros
- Factoring with Real Number Coefficients

Section 2-5, Day 1
HW

(3) $f(3)=0$ $(x-3)$ factor

$$\begin{array}{r|rrrr} 3 & 1 & 16 & -23 & -102 \\ & & 3 & 57 & 102 \\ \hline & 1 & 19 & 34 & 0 \end{array}$$

$f(x) = (x-3)(x^2+19x+34)$ $\sqrt{b^2-4ac}$ ↑ perfect sq

$(x-3)(x+17)(x+2)$

$$\boxed{x=3 \quad x=-17 \quad x=-2}$$

$2x^4 - x^3 - 18x^2 + 9x$

$x(2x^3 - x^2 - 18x + 9)$ $f(3)=0$

$x=0$ $3\rfloor\; 2 \quad -1 \quad -18 \quad 9$

degree 2

Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and nonreal). Some of these zeros may be repeated. $f(x) = 6x^7 \cdots$

7 zeros

Linear Factorization Theorem

If $f(x)$ is a polynomial function of degree $n > 0$, then $f(x)$ has precisely n linear factors and

$$f(x) = a(x - z_1)(x - z_2)\ldots(x - z_n)$$

where a is the leading coefficient of $f(x)$ and $z_1, z_2, \ldots, z_n$ are the complex zeros of $f(x)$. The z_i are not necessarily distinct numbers; some may be repeated.

Fundamental Polynomial Connections in Complex Case

The following statements about a polynomial function f are equivalent if k is a complex number:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$
2. k is a zero of the function f.
3. $x - k$ is a factor of $f(x)$.

complex number

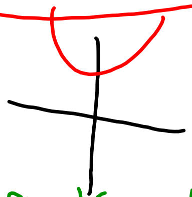
$x = 2i$

$x = 2i$ solution

zero

$(x - 2i)$ factor

No x-interept



$y = x^2 + 4$

X-intercepts are real solutions

Exploring Fundamental Polynomial Connections

Write the polynomial function in standard form, and identify the zeros of the function and the x-intercepts of its graph.

$f(x) = (x - 2i)(x + 2i)$

$f(x) = x^2 \cancel{+2ix} \cancel{-2ix} - 4(i)^2$

$x^2 - 4(-1)$

$x^2 + 4$

zeros $x - 2i = 0$ $\boxed{x = 2i}$

$x + 2i = 0$ $\boxed{x = -2i}$

X-intercepts: none

$f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

$f(x) = (x-5)(x^2 - 2i^2)$

$(x-5)(x^2+2)$

$f(x) = x^3 - 5x^2 + 2x - 10$

zeros:

$x = 5$

$x = \sqrt{2}i$

$x = -\sqrt{2}i$

X-intercepts

one

$x = 5$

$f(x) = (x - 3)(x - 3)(x + i)(x - i)$

$x^2 - i^2$

standard form

$x^4 - 6x^3 + 10x^2 - 6x + 9$

$(x^2 - 6x + 9)(x^2 + 1)$

$x^4 - 6x^3 + 9x^2 + x^2 - 6x + 9$

$x^4 - 6x^3 + 10x^2 - 6x + 9$

zeros:

$x = 3$ $x = 3$

$x = i$ $x = -i$

x-int:

$x = 3$

Write the polynomial in standard form and identify the zeros of the function and the x-intercepts of its graph.

$$f(x) = (x - 3i)(x + 3i)$$

x^2+9

Complex Conjugates Zeros

Suppose that $f(x)$ is a polynomial function with real coefficients. If a and b are real numbers with $b \neq 0$ and $a+bi$ is a zero of $f(x)$, then its complex conjugate $a-bi$ is also a zero of $f(x)$.

$2-3i$ $3i$ $2i$ i

$2+3i$ $-3i$ $-2i$ $-i$

Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include -3, 4 and $2\cancel{-}i$.

$$-3,\ 4,\ -2i,\ 2i$$

$$(x+3)(x-4)\underline{(x+2i)(x-2i)}$$

$$(x^2-x-12)(x^2+4)$$

$$x^4-x^3\underline{-12x^2+4x^2}-4x-48$$

$$x^4-x^3-8x^2-4x-48$$