Complex Zeros and the Fundamental Theorem of Algebra

- Two Major Theorems
- Complex Conjugate Zeros
- Factoring with Real Number Coefficients

Section 2-5, Day 1
HW

Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and nonreal). Some of these zeros may be repeated.

Linear Factorization Theorem

If $f(x)$ is a polynomial function of degree $n > 0$, then $f(x)$ has precisely n linear factors and

$$f(x) = a(x - z_1)(x - z_2)...(x - z_n)$$

where a is the leading coefficient of $f(x)$ and $z_1, z_2, ..., z_n$ are the complex zeros of $f(x)$. The z_i are not necessarily distinct numbers; some may be repeated.

Fundamental Polynomial Connections in Complex Case

The following statements about a polynomial function f are equivalent if k is a complex number:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$
2. k is a zero of the function f.
3. $x - k$ is a factor of $f(x)$.

Exploring Fundamental Polynomial Connections

Write the polynomial function in standard form, and identify the zeros of the function and the x-intercepts of its graph.

$f(x) = (x - 2i)(x + 2i)$

$f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

$f(x) = (x - 3)(x - 3)(x + i)(x - i)$

Write the polynomial in standard form and identify the zeros of the function and the x-intercepts of its graph.

$$f(x) = (x - 3i)(x + 3i)$$

Complex Conjugates Zeros

Suppose that $f(x)$ is a polynomial function with real coefficients. If a and b are real numbers with $b \neq 0$ and $a + bi$ is a zero of $f(x)$, then its complex conjugate $a - bi$ is also a zero of $f(x)$.

Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include -3, 4 and $2 - i$.

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include those listed.

$$1,\ 3i \text{ and } -3i$$

Finding a Polynomial from Given Zeros

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros include $x = 1$, $x = 1 + 2i$, $x = 1 - i$.

Write a polynomial function of minimum degree in standard form with real coefficients whose zeros and their multiplicities include those listed.

1 (multiplicity 2) -2 (multiplicity 3)

Factoring a Polynomial with Complex Zeros

Find all zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$, and write f(x) in its linear factorization.

Find all of the zeros and write a linear factorization of the function

$$f(x) = x^4 + x^3 + 5x^2 - x - 6$$

Finding Complex Zeros

The complex number $z = 1 - 2i$ is a zero of

$$f(x) = 4x^4 + 17x^2 + 14x + 65.$$

Find the remaining zeros of f(x) and write it in its linear factorization.

Using the given zero, find all the zeros and write the linear factorization of f(x)

$1 + i$ is a zero of $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$

Factors of Polynomial with Real Coefficien

Every polynomial function with real coefficients can be written as a product of linear factors and irreducible quadratic factors, each with real coefficients.

Factoring a Polynomial

Write $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$ as a product of linear and irreducible quadratic factors, each with real coefficients.

Write the function as a product of linear and irreducible quadratic factors all with real coefficients.

$$f(x) = x^3 - x^2 - x - 2$$