Real Zeros of Polynomial Functions

- Long Division and the Division Algorithm
- Remainder and Factor Theorems
- Synthetic Division
- Rational Zeros Theorem
- Upper and Lower Bounds

Section 2-4
MathXL...due Tuesday at midnight

$$9\,\overline{)\,67809}$$

Division Algorithm for Polynomials

Let $f(x)$ and $d(x)$ be polynomials with the degree of f greater than or equal to the degree of d, and $d(x) \neq 0$. Then there are unique polynomials $q(x)$ and $r(x)$, called the **quotient** and **remainder**, such that

$$f(x) = d(x) \cdot q(x) + r(x)$$

where either $r(x) = 0$ or the degree of r is less than the degree of d.

$$f(x) = d(x) \cdot q(x) + r(x)$$

dividend = (divisor)(quotient) + remainder

Divide f(x) by d(x) and write a summary statement in polynomial form and fraction form.

$f(x) = x^2 - 2x + 3$ $\qquad$ $d(x) = x - 1$

Divide f(x) by d(x) and write a summary statement in polynomial form and fraction form.

$f(x) = 2x^3 - 2x^2 + x + 3$ $\qquad$ $d(x) = x^2 + x + 3$

Using Polynomial Long Division

Use long division to find the quotient and remainder when $2x^4 - x^3 - 2$ is divided by $2x^2 + x + 1$. Write a summary statement in both polynomial and fraction form.

$$\begin{array}{r} x^2 - x \\ 2x^2+x+1 \overline{)\, 2x^4 - x^3 + 0x^2 + 0x - 2} \\ \underline{2x^4 + x^3 + x^2 } \\ -2x^3 - x^2 + 0x \\ \underline{-2x^3 - x^2 - x } \\ x - 2 \end{array}$$

Remainder

Remainder Theorem

If polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Using the Remainder Theorem

Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by...

a) $x - 2$

b) $x + 1$

c) $x + 4$

Are any of the above factors of $f(x)$?

$x+4 = 0$

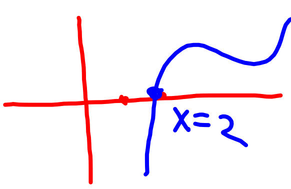
$x = -4$

$$
\begin{array}{r}
x^2 - 3x + 5 \\
x^2+1 \overline{\smash{)}\, x^4 - 3x^3 + 6x^2 - 3x + 5} \\
\underline{x^4 \qquad\qquad + x^2 \qquad\qquad\quad} \\
-3x^3 + 5x^2 - 3x \qquad \\
\underline{-3x^3 \qquad\quad - 3x \qquad} \\
5x^2 + 5 \\
\underline{5x^2 + 5}
\end{array}
$$

Fundamental Connections for Polynomial Functions

For a polynomial function f and a real number k the following statements are equivalent:

1. $x = k$ is a solution (or root) of the equation $f(x) = 0$ $x=2$
2. k is a zero of the function f. 2 is a zero
3. k is an x-intercept of the graph of $y = f(x)$. 2
4. $x - k$ is a factor of $f(x)$. $(x-2)$ is a factor



Using Synthetic Division

Divide $2x^3 - 3x^2 - 5x - 12$ by $x - 3$ using synthetic division and write a summary statement.

$x - c$

$x + c$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & \vert\ 0 \end{array}$$

$x - 3 = 0$

$x = 3$

$$2x^3 - 3x^2 - 5x - 12 = (x-3)(2x^2 + 3x + 4)$$

Divide using synthetic division, and write a summary statement in fraction form.

$$f(x) \quad \frac{x^3 - 5x^2 + 3x - 2}{x + 1}$$

$x + 1 = 0$

$x = -1$

$-1 \,\rfloor$

$R = -11$

$f(-1) = -11$

$x^3 - 5x^2 + 3x - 2$

$-1 - 5 - 3 - 2 = -11$

Find all the zero of $8x^3 - 30x^2 + 13x + 30$

given $x - 2$ is a factor

$x - 2 = 0$

$x = 2$

$$\begin{array}{r|rrrr} 2 & 8 & -30 & 13 & 30 \\ & & 16 & -28 & -30 \\ \hline & 8 & -14 & -15 & 0 \end{array}$$

$(x-2)(8x^2 - 14x - 15)$

↳ find zeros

Find all the zero of $2x^3 - x^2 - 41x - 20$

given $2x + 1$ is a factor

The Rational Root Theorem

For a polynomial function $Ax^n + Bx^{n-1} + Cx^{n-2} + \ldots + Z$, where A, B, C, ... Z are constants, let **q** be all of the positive and negative integer factors of **A** (the leading coefficient) and let **p** = all of the positive and negative integer factors of **Z** (the constant term). Then if there are any rational roots of the function, they are of the form $\pm p/q$ for any combination of p's and q's.

For example, for $f(x) = 6x^4 - 4x^3 + 9x^2 + 3,$

$q = 1, 2, 3, 6$ $p = 1, 3$

This is a complete list of any **possible** rational roots of f(x).

$$\frac{p}{q} = \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{3}{2}$$

P Q 6

3 1 1 6 2 3

$\frac{P}{Q}$ $\cancel{\frac{1}{1}}$ $\cancel{\frac{1}{6}}$ $\cancel{\frac{1}{2}}$ $\frac{1}{3}$ $\pm$

$\frac{3}{1}$ $\cancel{\frac{3}{6}}$ $\frac{3}{2}$ $\cancel{\frac{3}{3}}$ $\pm$

$\pm$ 1 $\frac{1}{6}$ $\frac{1}{2}$ $\frac{1}{3}$ $3, \frac{3}{2}$

Finding the Rational Zeros

Find the rational zeros of $f(x) = x^3 - 3x^2 + 1$

Finding the Rational Zeros

Find the rational zeros of $f(x) = 3x^3 + 4x^2 - 5x - 2$

Use the Rational Zeros Theorem to write a list of all potential rational zeros. Then determine which ones, if any, are zeros.

$$f(x) = 6x^3 - 5x - 1$$

Find all real zeros of the function, finding exact values whenever possible. Identify each zero as rational or irrational.

$$f(x) = 2x^4 - 7x^3 - 2x^2 - 7x - 4$$