Key

Calculus AB—Exam 1

Section I, Part A

Time: 55 minutes
Number of questions: 28

NO CALCULATOR MAY BE USED IN THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$y' = \frac{1}{2}x^4 + 2x^3$

$y'' = 2x^3 + 6x^2$

$2x^2(x+3) = 0$

$x = -3$

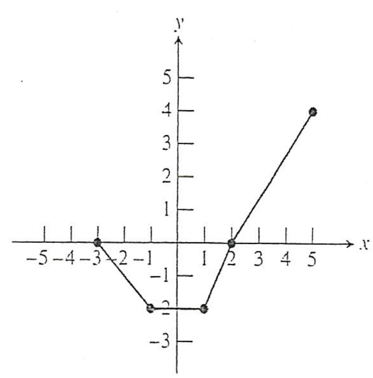
not an inflection pt

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{10}x^5 + \frac{1}{2}x^4 - \frac{3}{10}$?

(A) -4 (B) -3 (C) -1

(D) $-\frac{3}{10}$ (E) 0

2. The graph of a piecewise-linear function f, for $-3 \leq x \leq 5$, is shown. What is the value of $\int_{-3}^{5} f(x)\,dx$?



area of trapezoid (under axis)
-7

area of triangle
6

(A) 16 (B) 13 (C) 4

(D) 1 (E) -1

$$\int_2^3 x^{-3}\,dx$$

$$-\frac{1}{2}x^{-2}\Big|_2^3$$

$$-\frac{1}{2}\left[\frac{1}{x^2}\right]_2^3$$

$$-\frac{1}{2}\left[\frac{1}{9}-\frac{1}{4}\right]$$

$$-\frac{1}{2}\left[\frac{4}{36}-\frac{9}{36}\right]$$

$$-\frac{1}{2}\left[\frac{-5}{36}\right]=\frac{5}{72}$$

3. $\int_2^3 \frac{1}{x^3}\,dx =$

(A) $-\frac{5}{72}$ (B) $-\frac{5}{36}$ (C) $\frac{5}{144}$

(D) $\frac{5}{72}$ (E) $\ln\frac{27}{8}$

4. f is continuous for $a \le x \le b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f.

(B) $\lim_{x\to c} f(x) \neq f(c)$

(C) The graph of f has a cusp at $x = c$.

(D) $f(c)$ is undefined.

(E) None of the above

5. $\int_{\pi/2}^{x} \cos t\,dt =$

$\sin t\Big|_{\frac{\pi}{2}}^{x}$ $\sin x - \sin\frac{\pi}{2}$

$\sin x - 1$

(A) $-\sin x$ (B) $-\sin x - 1$

(C) $\sin x + 1$ (D) $\sin x - 1$

(E) $1 - \sin x$

$$x^3+2x^2y-4y=7$$

$$3x^2+2x^2\frac{dy}{dx}+4xy-4\frac{dy}{dx}=0$$

$$2x^2\frac{dy}{dx}-4\frac{dy}{dx}=-3x^2-4xy$$

$$\frac{dy}{dx}(2x^2-4)=-3x^2-4xy$$

$$\frac{dy}{dx}=\frac{-3x^2-4xy}{2x^2-4}\Bigg|_{(1,-3)}$$

$$\frac{-3(1)^2-4(1)(-3)}{2(1)^2-4}$$

$$\frac{-3+12}{-2}=-\frac{9}{2}$$

6. If $x^3 + 2x^2y - 4y = 7$, then when $x = 1$, $\frac{dy}{dx} =$

when $x=1$

$1+2y-4y=7$

$y=-3$

(A) $-\frac{9}{2}$. (B) 0.

(C) -8. (D) -3.

(E) $\frac{7}{2}$.

7. $\int_1^{e^2} \frac{x^3+1}{x}\,dx =$

$\int_1^{e^2}\left(x^2+\frac{1}{x}\right)dx$ $\frac{1}{3}x^3+\ln x\Big|_1^{e^2}$

(A) $\frac{1}{3}e^6 + \frac{8}{3}$ (B) $\frac{1}{3}e^6 + \frac{5}{3}$

$\frac{1}{3}e^6+2-$

$\left(\frac{1}{3}+0\right)$

(C) $\frac{1}{3}e^6 - \frac{1}{2e^2} + \frac{1}{6}$ (D) $\frac{1}{3}e^6 - \frac{1}{2e^4} + \frac{1}{6}$

(E) $\frac{1}{3}e^6 + \frac{7}{3}$

$\frac{1}{3}e^6+\frac{5}{3}$

8. Let f and g be differentiable functions with the following properties:

I. $f(x) < 0$ for all x

II. $g(5) = 2$

If $h(x) = \dfrac{f(x)}{g(x)}$ and $h'(x) = \dfrac{f'(x)}{g(x)}$, then $g(x) =$

(A) $\dfrac{1}{f'(x)}$. (B) $f(x)$.

(C) $-f(x)$. (D) 0.

(E) 2.

$$h'(x) = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{g(x)f'(x)}{(g(x))^2} - \frac{f(x)g'(x)}{(g(x))^2}$$

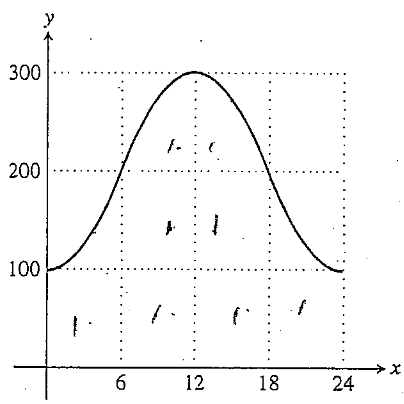
$$\frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{(g(x))^2} = \frac{f'(x)}{g(x)}$$

$\therefore f(x)\cdot g'(x) = 0$

$g'(x) = 0$

$g(x)$ is a constant

9. The production rate of cola, in thousands of gallons per hour, at the production plant on July 1 is shown in the graph. Of the following, which best approximates the total number of thousands of gallons of cola that were produced that day?



$(600)8 =$

4800

(A) 800 (B) 4200 (C) 4800

(D) 5000 (E) 5400

10. What is the instantaneous rate of change at $x = 3$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x + 1}$?

(A) $-\frac{17}{16}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{8}$

(D) $\frac{13}{16}$ (E) $\frac{17}{16}$

$$f'(x) = \frac{(x+1)(2x) - (x^2-2)(1)}{(x+1)^2}$$

$$f'(3) = \frac{4(6) - 7(1)}{16} = \frac{17}{16}$$

11. If f is a linear function and $0 < a < b$, then $\int_b^a f''(x)\,dx =$

f' is a constant

f'' is zero

(A) 0 (B) 2 (C) $\frac{ab}{2}$

(D) $m(a-b)$ (E) $\frac{a^2 - b^2}{2}$

12. If $f(x) = \begin{cases} \ln 3x, & 0 < x \le 3 \\ x \ln 3, & 3 < x \le 4 \end{cases}$, then $\lim_{x \to 3} f(x)$ is

$y = \ln(3)(3) = \ln 9$

$y = 3\ln 3$

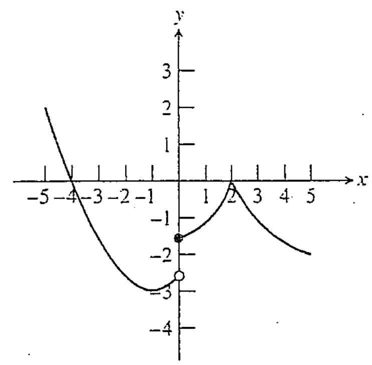
$3\ln 3 \neq \ln 9$

(A) ln 9. (B) ln 27. (C) 3 ln 3.

(D) 3 + ln 3. (E) nonexistent.

right limit ≠ left limit

13. The graph of the function f shown in the figure has a horizontal tangent at the point $(-1, -3)$ and a cusp at $(2, 0)$. For what values of x, $-5 < x < 5$, is f not differentiable?



(A) 0 only (B) 0 and 2 only (C) −1 and 0 only

(D) −1, 0, and 2 (E) −1 and 2 only

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 7t + 12$. For what value of t is the velocity of the particle zero?

$x'(t) = 2t - 7$

$2t - 7 = 0$

$t = \frac{7}{2}$

(A) 2.5 (B) 3 (C) 3.5

(D) 4 (E) 4.5

15. If $F(x) = \int_1^{x^2} \sqrt{t^2 + 3}\, dt$, then $F'(2) =$

$F'(x) = \sqrt{(x^2)^2+3} \cdot 2x$

(A) $4\sqrt{19}$ (B) $2\sqrt{19}$ (C) $4\sqrt{7}$

(D) $2\sqrt{7}$ (E) $\sqrt{7}$

$F'(2) = \sqrt{2^4+3}\ (2)(2)$

$= \sqrt{19}\,(4)$

16. If $f(x) = \cos e^{2x}$, then $f'(x) =$

(A) $\sin e^{2x}$ (B) $2 \sin e^{2x}$ (C) $-\sin e^{2x}$

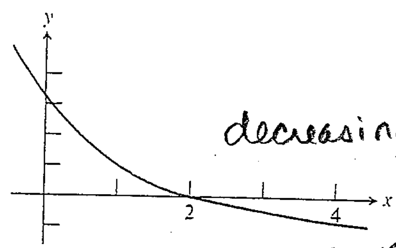
(D) $-2 \sin e^{2x}$ (E) $-2e^{2x} \sin e^{2x}$

$f(x) = \cos e^{2x}$

$f'(x) = -\sin e^{2x} \cdot e^{2x} \cdot 2$

$= -2e^{2x} \sin e^{2x}$

17. The graph of a twice-differentiable function f is shown. Which of the following is true?



$f(2) = 0$

decreasing $\rightarrow$ $f'(2) < 0$

concave up $\rightarrow$ $f''(2) > 0$

(A) $f(2) < f'(2) < f''(2)$ (B) $f(2) < f''(2) < f'(2)$

(C) $f'(2) < f(2) < f''(2)$ (D) $f''(2) < f(2) < f'(2)$

(E) $f''(2) < f'(2) < f(2)$

18. An equation of the line tangent to the graph of $y = 3x - \cos x$ at $x = 0$ is

(A) $y = 2x$ (B) $y = 2x - 1$ (C) $y = 3x + 1$

(D) $y = 3x - 1$ (E) $y = 4x$

$y' = 3 + \sin x$

$y'(0) = 3 + \sin 0$

$= 3$

$y = 3(0) - \cos 0$

$y = -1$

point $(0, -1)$ slope $= 3$

$y + 1 = 3x$

$y = 3x - 1$

19. If $f''(x) = (x-1)(x+2)^3(x-4)^2$, then the graph of f has inflection points when $x =$

(A) -2 only (B) 1 only (C) 1 and 4 only
(D) -2 and 1 only (E) -2, 1, and 4 only

20. What are all values of k for which $\int_{-2}^{k} x^5\,dx = 0$?

(A) -2 (B) 0 (C) 2
(D) -2 and 2 (E) -2, 0, and 2

at $K=-2$ and:

$\frac{1}{6}x^6 \Big|_{-2}^{K} = 0$

$\frac{1}{6}[K^6 - 2^6] = 0$

$K = 2$

21. If $dy/dt = my$ and m is a nonzero constant, then y could be

(A) $4e^{mty}$. (B) $4e^{mt}$. (C) $e^{mt} + 4$.
(D) $mty + 4$. (E) $\frac{m}{2}y^2 + 4$.

$y = 4e^{mt}$

$\frac{dy}{dt} = 4e^{mt} \cdot m$

$= 4me^{mt}$

$= m\underline{4e^{mt}}$

$= my$

22. The function f is given by $f(x) = -x^6 + x^3 - 2$. On which of the following intervals is f decreasing?

(A) $(-\infty, 0)$ (B) $\left(-\infty, -\sqrt[3]{\frac{1}{2}}\right)$ (C) $\left(0, \sqrt[3]{\frac{1}{2}}\right)$
(D) $(0, \infty)$ (E) $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$

$f(x) = -x^6 + x^3 - 2$

$f'(x) = -6x^5 + 3x^2$

$-6x^5 + 3x^2 = 0$

$-6x^5 + 3x^2 < 0$

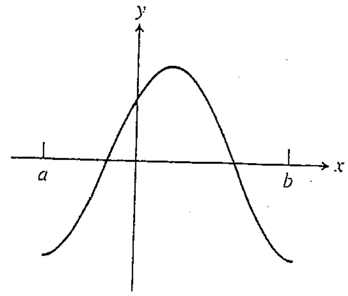
$6x^5 > 3x^2$ divide by x^2 always positive

$2x^3 > 1$

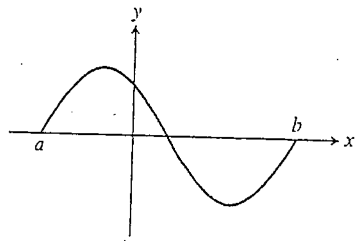
$x^3 > \frac{1}{2}$

$x > \sqrt[3]{\frac{1}{2}}$

23. The graph of f is shown in the figure below. Which of the following could be the graph of the derivative of f?



(A)



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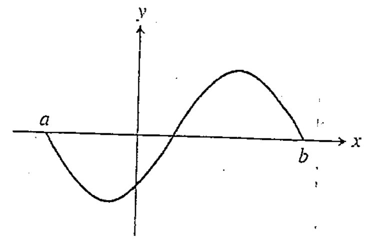
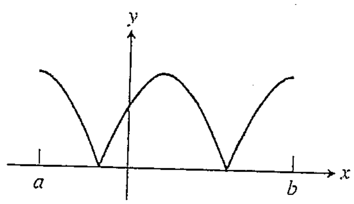
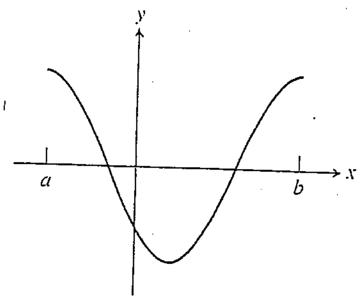


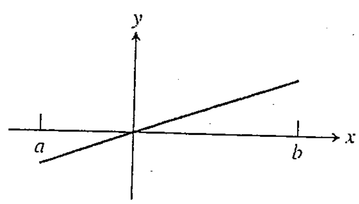
(C)



(D)



(E)



24. The minimum acceleration attained on the interval $0 \leq t \leq 4$ by the particle whose velocity is given by $v(t) = t^3 - 4t^2 - 3t + 2$ is

(A) -16. (B) -10. (C) -8.

(D) $-\frac{25}{3}$. (E) -3.

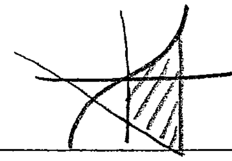
$a(t) = 3t^2 - 8t - 3 \leftarrow$ function to minimi

$a'(t) = 6t - 8$

$t = \frac{4}{3}$

$a(\frac{4}{3}) = 3(\frac{4}{3})^2 - 8(\frac{4}{3}) - 3$

$= -\frac{25}{3}$



$\int_0^2 (x^3 - (-x-1))\,dx$

$\int_0^2 (x^3+x+1)\,dx$

$\frac{1}{4}x^4 + \frac{1}{2}x^2 + x \Big|_0^2$

$\frac{1}{4}(16) + \frac{1}{2}(4) + 2$

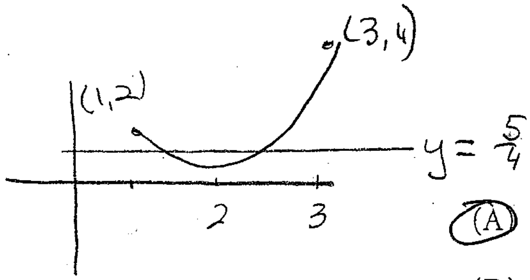
$= 8$

25. What is the area of the region between the graphs of $y = x^3$ and $y = -x - 1$ from $x = 0$ to $x = 2$?

(A) 0 (B) 4 (C) 5
(D) 8 (E) 10

26. The function f is continuous on the closed interval $[1, 3]$ and has the values given in the table. The equation $f(x) = \frac{5}{4}$ must have at least two solutions in the interval $[1, 3]$ if $k =$

x	1	2	3
$f(x)$	2	k	4



Function must go below $\frac{5}{4}$

(A) $\frac{1}{4}$. (B) $\frac{3}{2}$. (C) 2.
(D) $\frac{9}{4}$. (E) 3.

27. What is the average value of $y = x^3\sqrt{x^4 + 9}$ on the interval $[0, 2]$?

(A) $\frac{98}{3}$ (B) $\frac{49}{3}$ (C) $\frac{125}{12}$
(D) $\frac{147}{8}$ (E) $\frac{49}{6}$

$\frac{1}{2}\int_0^2 x^3\sqrt{x^4+9}$

$u = x^4 + 9$

$du = 4x^3$

$\frac{1}{2}\cdot\frac{1}{4}\int_0^2 4x^3\sqrt{x^4+9}$

$u(0) = 9 \quad u(2) = 25$

$\frac{1}{8}\int_9^{25} u^{\frac{1}{2}}\,du$

$\frac{1}{8}\cdot\frac{2}{3}u^3\Big|_9^{25}$

$\frac{1}{12}\left[125 - 27\right]$

$\frac{1}{12}(98) = \frac{49}{6}$

28. If $f(x) = \tan 3x$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{4}{3}$ (B) 4 (C) 6
(D) 12 (E) $6\sqrt{3}$

$f(x) = \tan 3x$

$f'(x) = 3\sec^2 3x$

$f'\left(\frac{\pi}{9}\right) = 3\sec^2\frac{\pi}{3}$

$= 3(2)^2$

$= 12$

Key

Calculus AB—Exam 1
Section I, Part B

Time: 50 Minutes
Number of Questions: 17

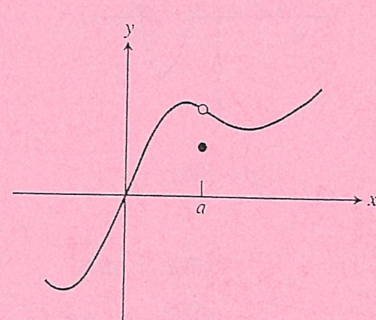
A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS PART OF THE EXAMINATION.

<u>Directions:</u> Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

<u>In this test:</u>

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The graph of a function f is shown. Which of the following statements about f is false?



(A) $\lim_{x\to a} f(x)$ exists.

(B) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

(C) $f(a)$ exists.

(D) f is continuous at $x = a$.

(E) f has a relative minimum at $x = a$.

$f'(x) = g'(x)$

y_1 $f'(x) = 6e^{3x}$ int

y_2 $g'(x) = 15x^2$

$x = -.366$

30. Let $f(x) = 2e^{3x}$ and $g(x) = 5x^3$. At what value of x do the graphs of f and g have parallel tangents?

(A) -0.445 (B) -0.366 (C) -0.344

(D) -0.251 (E) -0.165

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{dV}{dt} = 3s^2(0.2)$

$\frac{dV}{dt} = 3\,\frac{SA}{6}(0.2)$

$\frac{dV}{dt} = \frac{1}{2}(0.2)\,SA$

$= .1SA$

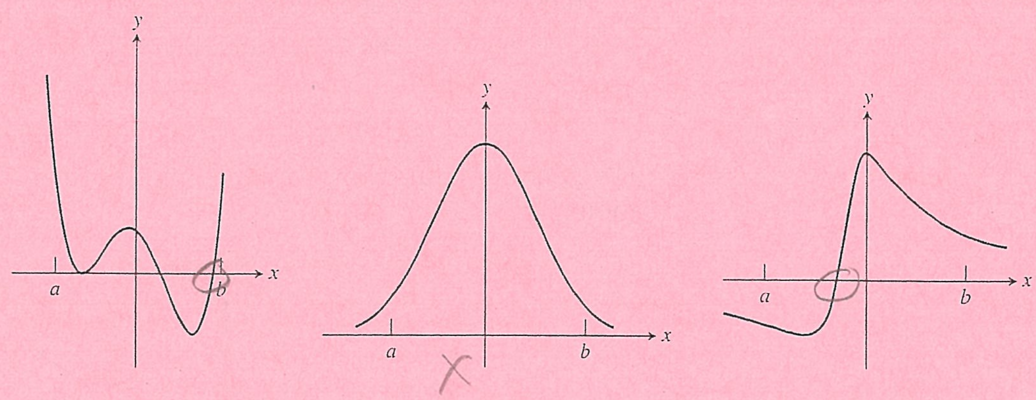
edge

31. The side of a cube is increasing at a constant rate of 0.2 centimeter per second. In terms of the surface area S, what is the rate of change of the volume of the cube, in cubic centimeters per second?

(A) $0.1S$ (B) $0.2S$ (C) $0.6S$

(D) $0.04S$ (E) $0.008S$

32. The graphs of the derivatives of the functions f, g, and h are shown. Which of the functions have a relative minimum on the open interval $a < x < b$?



$y = f'(x)$ $y = g'(x)$ $y = h'(x)$

(A) f only (B) g only (C) h only

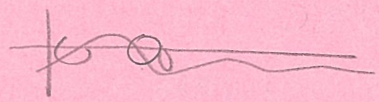
(D) f and h only (E) f, g, and h

33. The first derivative of the function f is given by $f'(x) = \dfrac{\sin^2 x}{x} - \dfrac{2}{9}$. How many critical values does f have on the open interval $(0, 10)$?

zeros of f'

(A) One (B) Two (C) Three

(D) Four (E) Six

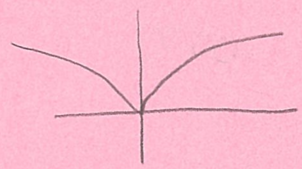


34. Let f be the function given by $f(x) = x^{2/3}$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.

II. f is differentiable at $x = 0$.

III. f has an absolute minimum at $x = 0$.



(A) I only (B) II only (C) III only

(D) I and II only (E) I and III only

35. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_{-1}^{2} f(3x)\,dx =$

(A) $3F(2) - 3F(-1)$ (B) $\frac{1}{3}F(2) - \frac{1}{3}F(-1)$

(C) $F(6) - F(-3)$ (D) $3F(6) - 3F(-3)$

(E) $\frac{1}{3}F(6) - \frac{1}{3}F(-3)$

$\frac{1}{3}\int_{-1}^{2} f(3x)\,3\,dx$

$\frac{1}{3}(F(6) - \frac{1}{3}F(-3))$

36. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^3 - a^3}{a^6 - x^6}$ is

(A) nonexistent. (B) 0. (C) $-\dfrac{1}{2a^3}$.

(D) $-\dfrac{1}{a^3}$. (E) $\dfrac{1}{2a^3}$.

$\dfrac{x^3 - a^3}{(a^3 - x^3)(a^3 + x^3)}$ $\dfrac{-1}{a^3 + x^3}$

$\dfrac{-1}{2a^3}$

37. Population P grows according to the equation $dP/dt = kP$, where k is a constant and t is measured in years. If the population doubles every 12 years, then the value of k is approximately

(A) 3.585. (B) 1.792. (C) 0.693.

(D) 0.279. (E) 0.058.

$\frac{dP}{dt} = kP$

$y = y_0 e^{kt}$

$2 = e^{k(12)}$

$\frac{\ln 2}{12} = \frac{12k}{12}$

$k = .0577$

38. The function f is continuous on the closed interval $[1, 9]$ and has the values given in the table. Using the subintervals $[1, 3]$, $[3, 6]$, and $[6, 9]$, what is the value of the trapezoidal approximation of $\int_1^9 f(x)\,dx$?

x	1	3	6	9
$f(x)$	15	25	40	30

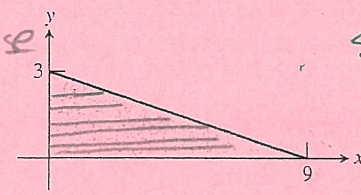
$2(20) + 3(32.5) + 3(35)$

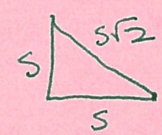
(A) 110 (B) 150 (C) 175

(D) 242.5 (E) 262.5

39. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 3y = 9$, as shown in the figure. If cross sections of the solid perpendicular to the y-axis are isoceles right triangles with the hypotenuses in the xy-plane, what is the volume of the solid?

X-values are hypotenuse





$x = 9 - 3y$

$\text{leg} = \frac{9-3y}{\sqrt{2}}$

$\text{area} = \frac{1}{2}\,\text{leg}^2$

$\text{area} = \frac{1}{2}\left(\frac{(9-3y)^2}{2}\right)$

$\int_0^3 \frac{(9-3y)^2}{4}\,dy$

(A) 6.75 (B) 13.5 (C) 15.188

(D) 20.25 (E) 40.5

40. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

(A) $y = -x - 1.031$ (B) $y = -x - 0.836$

(C) $y = -x + 0.836$ (D) $y = -x + 0.934$

(E) $y = -x + 1.031$

y_1 $f'(x) = 6x^5 - 4x^3$

y_2 $6x^5 - 4x^3 = -1$

intersectin

$x = -.9335504$

put in original find y

$y = -.0975886$

$y + .0975886 = -1(x + .93355)$

$y = -x - 1.031$

41. Let $F(x)$ be an antiderivative of $\dfrac{2(\ln x)^4}{3x}$. If $F(2) = 0$, then $F(8) =$

(A) 5.163. (B) 0.860. (C) 0.184.

(D) 0.180. (E) 0.004.

$\frac{2}{3}\frac{(\ln x)^4}{x}$

$\frac{2}{3} \cdot \frac{1}{5}(\ln x)^5 + C$

$\frac{2}{15}(\ln x)^5 + C$

$\frac{2}{15}(\ln 2)^5 + C = 0$

$C = -.02133$

$F(8) =$

$\frac{2}{15}(\ln 8)^5 - .02133$

42. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - x - 12)\,g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 4$.

(B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 4$.

(C) f has a relative maximum at $x = 3$ and a relative minimum at $x = -4$.

(D) f has a relative minimum at $x = 3$ and a relative maximum at $x = -4$.

(E) It cannot be determined if f has any relative extrema.

$f'(x) =$

$(x-4)(x+3)\underset{g(x)}{(-)}$

$-$ $+$ $-$

-3 4

43. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

(A) A is always increasing.

(B) A is always decreasing.

(C) A is increasing only when $l > w$.

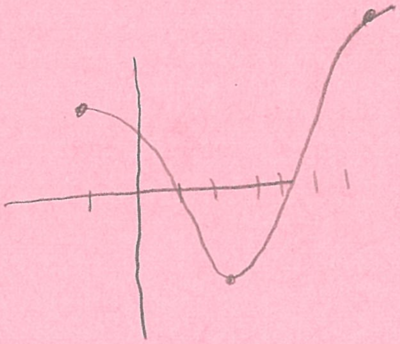
(D) A is increasing only when $l < w$.

(E) A remains constant.

$A = l \cdot w$

$\frac{dA}{dt} = l\frac{dw}{dt} + w \cdot \frac{dl}{dt}$

$\frac{dA}{dt} = l(2) + w(-2)$



44. Let f be a function that is differentiable on the open interval $(-3, 7)$. If $f(-1) = 4$, $f(2) = -5$, and $f(6) = 8$, which of the following must be true?

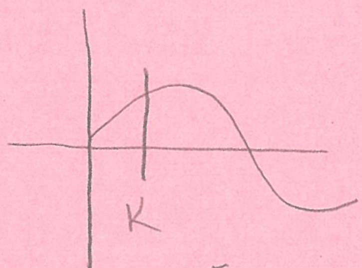
I. f has at least two zeros.

II. f has a relative minimum at $x = 2$.

III. For some c, $2 < c < 6$, $f(c) = 4$.

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III



45. If $0 \leq k \leq \frac{\pi}{2}$ and the area under the curve $y = \sin x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.75, then $k =$

(A) 1.318. (B) 0.848. (C) 0.723.

(D) 0.533. (E) 0.253.

$$\int_{K}^{\frac{\pi}{2}} \sin x \, dx = .75$$

$$-\cos\frac{\pi}{2} + \cos K = 0.75$$

$$+\cos K = 0.75 + \cos\frac{\pi}{2}$$

$$\cos K = 0.75$$

$$K = \cos^{-1}(0.75)$$

$$K = .723$$