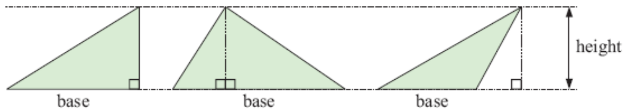
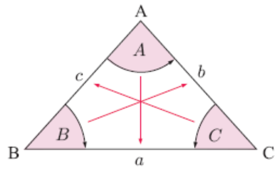
# F AREAS OF TRIANGLES

If we know the base and height measurements of a triangle, we can calculate the area using **area** $= \frac{1}{2}$ **base** $\times$ **height**.

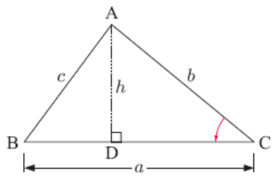


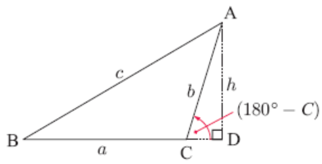

## THE AREA OF A TRIANGLE FORMULA

Suppose triangle ABC has angles of size $A$, $B$, and $C$, and the sides opposite these angles are labelled $a$, $b$, and $c$ respectively.




Any triangle that is not right angled must be either acute or obtuse. In either case we construct a perpendicular from A to D on BC (extended if necessary).




Using right angled trigonometry:

$$\sin C = \frac{h}{b} \qquad\qquad \sin(180^\circ - C) = \frac{h}{b}$$

$$\therefore\ h = b\sin C \qquad\qquad \therefore\ h = b\sin(180^\circ - C)$$

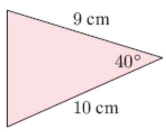
$$\therefore\ h = b\sin C$$

So, area $= \frac{1}{2}ah$ gives $\boldsymbol{A = \frac{1}{2}ab\sin C}$.

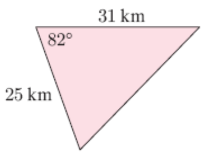
Using different altitudes we can show that the area is also $\frac{1}{2}bc\sin A$ or $\frac{1}{2}ac\sin B$.

**1** Find the area of:

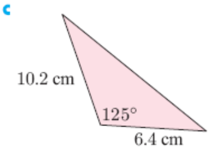
**a**



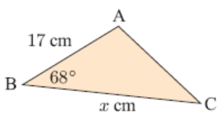

**b**




**c**




**2** If triangle ABC has area 150 cm$^2$, find the value of $x$:




**3** Calculate the area of:

**a** an isosceles triangle with equal sides of length 21 cm and an included angle of 49°

**b** an equilateral triangle with sides of length 57 cm.

**4** A parallelogram has adjacent sides of length 4 cm and 6 cm. If the included angle measures 52°, find the area of the parallelogram.

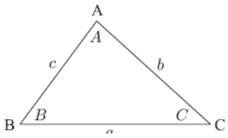
**5** A rhombus has sides of length 12 cm and an angle of 72°. Find its area.

# G THE COSINE RULE

The **cosine rule** involves the sides and angles of any triangle. The triangle does not need to contain a right angle.

In any $\triangle$ABC with sides $a$, $b$, and $c$ units in length, and opposite angles $A$, $B$, and $C$ respectively:

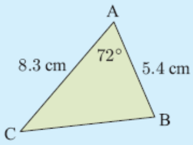
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{or} \quad b^2 = a^2 + c^2 - 2ac\cos B$$
$$\text{or} \quad c^2 = a^2 + b^2 - 2ab\cos C$$




## USING THE COSINE RULE

If we know the length of **two sides** of a triangle and the size of the **included angle** between them, we can use the cosine rule to find the third side.

Ex. Find the length BC:

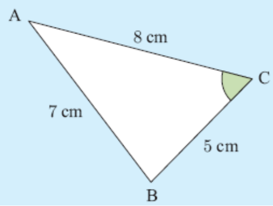



If we know all **three sides** of a triangle, we can rearrange the cosine rule formulae to find any of the angles:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

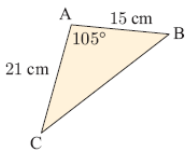
We use the **inverse** cosine ratio $\cos^{-1}$ to evaluate the angle.

Ex. In triangle ABC, AB = 7 cm, BC = 5 cm, and CA = 8 cm. Find the measure of angle BCA.

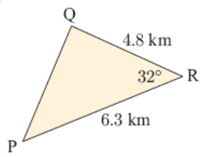



**1** Find the length of the remaining side in the given triangle:

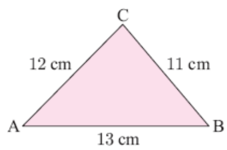
**a**




**b**

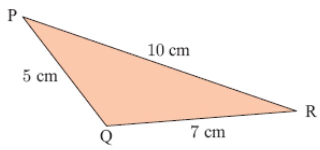



**2** Find the measure of all angles of:

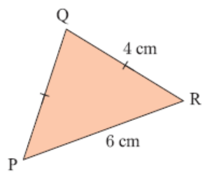



**3** Find the measure of obtuse angle PQR:

**a**




**b**




**4** **a** Find the smallest angle of a triangle with sides 11 cm, 13 cm, and 17 cm.

**b** Find the largest angle of a triangle with sides 4 cm, 7 cm, and 9 cm.