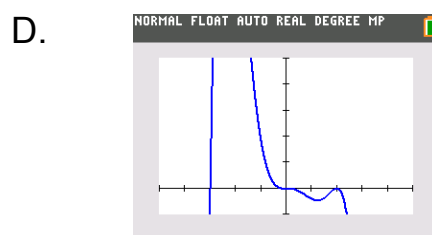
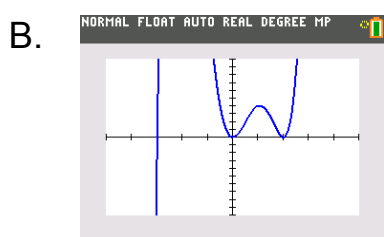
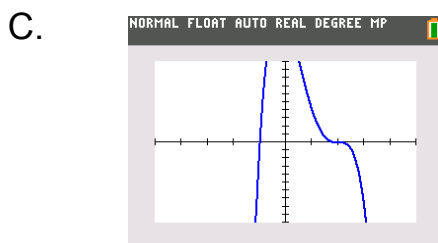
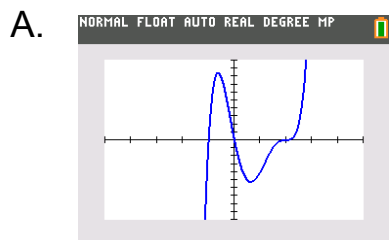


Given $f(x) = -x^3(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

b. Which of the below graphs would model $f(x)$?



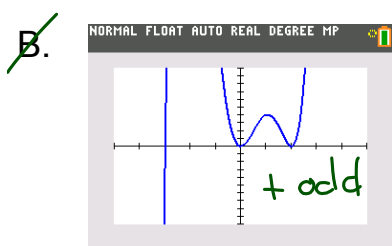
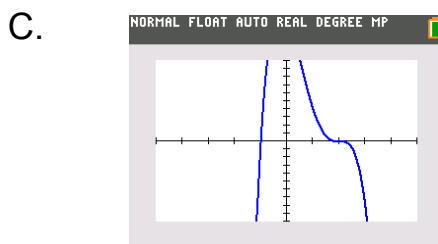
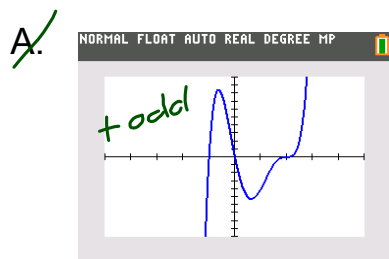
Given $f(x) = -x^3(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

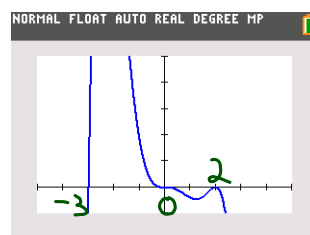
$0 \text{ m. } 3, 2 \text{ m. } 2, -3 \text{ m. } 1$

neg. deg 6 ↙ ↘

b. Which of the below graphs would model $f(x)$?



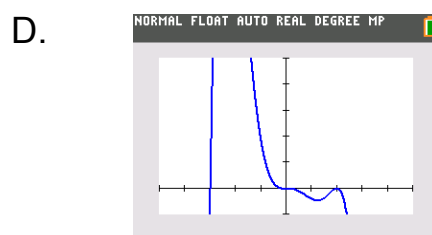
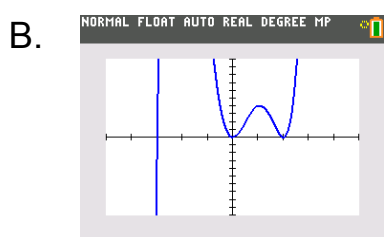
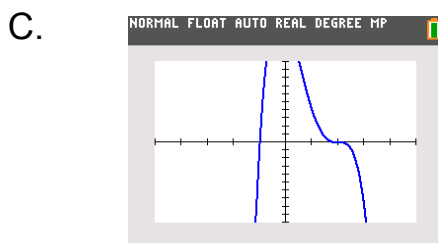
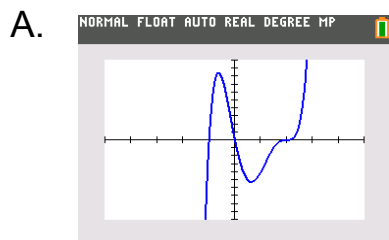
D.



Given $f(x) = x^2(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

b. Which of the below graphs would model $f(x)$?

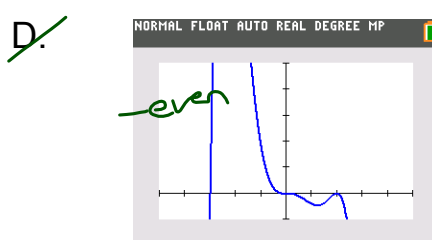
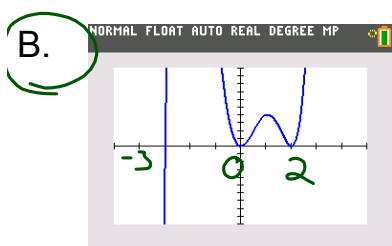
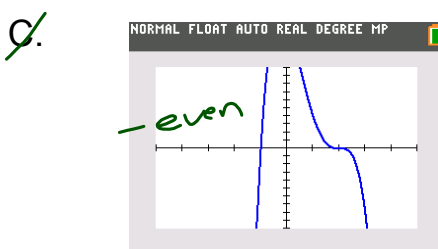
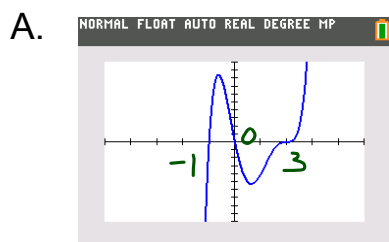


Given $f(x) = x^2(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

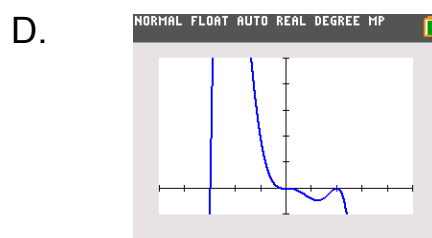
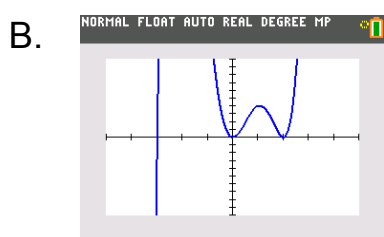
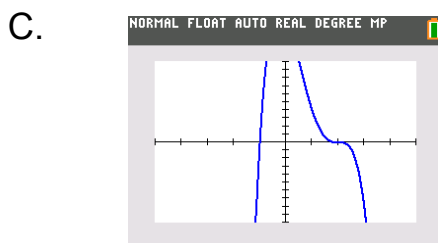
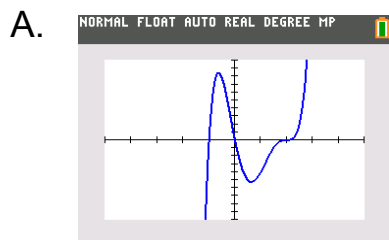
0 m. 2, 2 m. 2, -3 m. 1 pos. deg 5

b. Which of the below graphs would model $f(x)$?



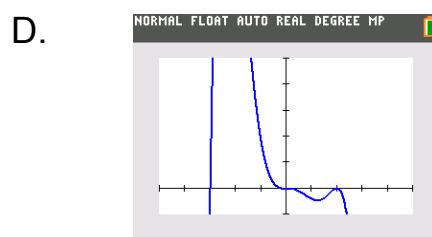
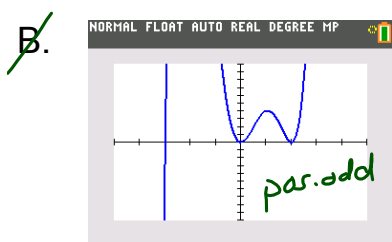
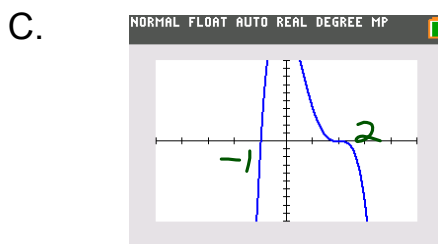
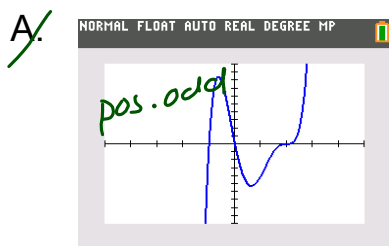
Given $f(x) = -2(x+1)(x-2)^3$

- Identify the roots and list their multiplicities.
- Which of the below graphs would model $f(x)$?



Given $f(x) = -2(x+1)(x-2)^3$

- Identify the roots and list their multiplicities.
 -1 m.1, 2 m.3 neg. deg 4
- Which of the below graphs would model $f(x)$?



Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = x^3 + 4x^2 + 7x - 9$$

$$d(x) = x + 3$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 2x^4 - 5x^3 + 7x^2 - 3x + 1$$

$$d(x) = x - 3$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = x^3 + 4x^2 + 7x - 9$$

$$d(x) = x + 3$$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 7 & -9 \\ & \downarrow & -3 & -3 & -12 \\ \hline & 1 & 1 & 4 & -21 \end{array}$$

$$x^2 + x + 4 \quad r. -21$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 2x^4 - 5x^3 + 7x^2 - 3x + 1$$

$$d(x) = x - 3$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 7 & -3 & 1 \\ & \downarrow & 6 & 3 & 30 & 81 \\ \hline & 2 & 1 & 10 & 27 & 82 \end{array}$$

$$2x^3 + x^2 + 10x + 27 \quad r. 82$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 9x^3 + 7x^2 - 3x$$

$$d(x) = x - 10$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 5x^4 - 3x + 1$$

$$d(x) = x - 4$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 9x^3 + 7x^2 - 3x$$

$$d(x) = x - 10$$

$$\begin{array}{r|rrrr} 10 & 9 & 7 & -3 & 0 \\ & \downarrow & 90 & 970 & 9670 \\ \hline & 9 & 97 & 967 & 9670 \end{array}$$

$$9x^2 + 97x + 967 \text{ r. } 9670$$

Divide $f(x)$ by $d(x)$ using synthetic division to find the quotient and remainder.

$$f(x) = 5x^4 - 3x + 1$$

$$d(x) = x - 4$$

$$\begin{array}{r|rrrrr} 4 & 5 & 0 & 0 & -3 & 1 \\ & \downarrow & 20 & 80 & 320 & 1268 \\ \hline & 5 & 20 & 80 & 317 & 1269 \end{array}$$

$$5x^3 + 20x^2 + 80x + 317 \text{ r. } 1269$$

Determine a polynomial of minimum degree with real coefficients that has roots $1/4$, $1 - 2i$ and a leading coefficient of 2. Write it in standard form.

Determine a polynomial of minimum degree with real coefficients that has roots $1/4$, $1 - 2i$ and a leading coefficient of 2. Write it in standard form.

$$\begin{aligned}
 & x = \frac{1}{4} \quad x = 1 - 2i \quad x = 1 + 2i \\
 & (4x - 1)(x - 1 + 2i)(x - 1 - 2i) \\
 & \quad \quad \quad x^2 - x - 2x + 1 + 2x - 2i + 2x - 2i + 4i^2 \\
 & (4x - 1)(x^2 - 2x + 5) \\
 & 4x^3 - 8x^2 + 20x - x^2 + 2x - 5 \\
 & \quad \quad \quad \frac{4x^3}{2} - \frac{9x^2}{2} + \frac{22x}{2} - \frac{5}{2} \\
 & \quad \quad \quad 2x^3 - \frac{9}{2}x^2 + 11x - \frac{5}{2}
 \end{aligned}$$

Determine a polynomial of minimum degree with real coefficients that has roots $-1/2$, $2 + 3i$ and a leading coefficient of 4. Write it in standard form.

Determine a polynomial of minimum degree with real coefficients that has roots $-1/2$, $2 + 3i$ and a leading coefficient of 4. Write it in standard form.

$$\begin{aligned}
 & x = -\frac{1}{2} \quad x = 2 + 3i \quad x = 2 - 3i \\
 & (2x + 1) \left\{ (x - 2 - 3i)(x - 2 + 3i) \right. \\
 & \quad \left. x^2 - 2x + 3xi - 2x + 4 - 6i - 3xi + 6i + 9i^2 \right.
 \end{aligned}$$

$$(2x + 1)(x^2 - 4x + 13)$$

$$2x^3 - 8x^2 + 26x + x^2 - 4x + 13$$

$$2(2x^3 - 7x^2 + 22x + 13)$$

$$4x^3 - 14x^2 + 44x + 26$$

Determine if $x + 5$ is a factor of $f(x) = 3x^3 + 2x^2 + x - 5$

Show your work and explain your answer.

Determine if $x - 1$ is a factor of $f(x) = x^3 - x^2 + x - 1$

Show your work and explain your answer.

Determine if $x + 5$ is a factor of $f(x) = 3x^3 + 2x^2 + x - 5$

Show your work and explain your answer.

$$f(-5) = 3(-5)^3 + 2(-5)^2 + (-5) - 5 = -335$$

no ... remainder is not 0.

Determine if $x - 1$ is a factor of $f(x) = x^3 - x^2 + x - 1$

Show your work and explain your answer.

$$f(1) = 1^3 - 1^2 + 1 - 1 = 0$$

yes ... remainder is 0 !!

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^3 - 5x - 1$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^4 - x^3 - 6x^2 - x - 12$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^3 - 5x - 1$$

$$p: 1 \quad \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

$$q: 1, 2, 3, 6$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^4 - x^3 - 6x^2 - x - 12$$

$$p: 1, 2, 3, 4, 6, 12$$

$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$$

$$q: 1, 2, 3, 6$$

$$\pm 2, \pm \frac{2}{3}$$

$$\pm 4, \pm \frac{4}{3}$$

$$\pm 3, \pm \frac{3}{2}$$

$$\pm 6, \pm 12$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 2x^3 - x^2 - 9x + 9$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 4x^3 + 7x^2 - x + 6$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 2x^3 - x^2 - 9x + 9$$

$$P: 1, 3, 9 \quad \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$$

$$Q: 1, 2$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 4x^3 + 7x^2 - x + 6$$

$$P: 1, 2, 3, 6 \quad \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6$$

$$Q: 1, 2, 4$$

Given $f(-1) = 0$, find all remaining zeros of $f(x) = x^3 - 2x^2 - 13x - 10$

Given $f(1) = 0$, find all remaining zeros of $f(x) = 3x^3 + 4x^2 - 5x - 2$

Given $f(-1) = 0$, find all remaining zeros of $f(x) = x^3 - 2x^2 - 13x - 10$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -13 & -10 \\ & \downarrow & -1 & 3 & 10 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$$x^2 - 3x - 10$$

$$x^2 - 3x - 10$$

$$(x-5)(x+2)$$

$$x = 5, -2, -1$$

Given $f(1) = 0$, find all remaining zeros of $f(x) = 3x^3 + 4x^2 - 5x - 2$

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -5 & -2 \\ & \downarrow & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$$

$$3x^2 + 7x + 2$$

$$3x^2 + 7x + 2$$

$$(3x+1)(x+2)$$

$$x = -\frac{1}{3}, -2, 1$$

Given $f(2)=0$, find all remaining zeros of $f(x) = x^3 - 3x - 2$

Given $f(3)=0$, find all remaining zeros of $f(x) = 2x^3 - 3x^2 - 5x - 12$

Given $f(2)=0$, find all remaining zeros of $f(x) = x^3 - 3x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & \downarrow & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$x^2 + 2x + 1$$

$$x^2 + 2x + 1$$

$$(x+1)(x+1)$$

$$\boxed{x = -1 \text{ m. 2. }, 2}$$

Given $f(3)=0$, find all remaining zeros of $f(x) = 2x^3 - 3x^2 - 5x - 12$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

$$2x^2 + 3x + 4$$

$$2x^2 + 3x + 4$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$x = \frac{-3 \pm \sqrt{-23}}{4} = \frac{-3 \pm i\sqrt{23}}{4}$$

Given $4i$ is a zero of $f(x) = x^4 + 13x^2 - 48$
find all remaining zeros.

Given $4i$ is a zero of $f(x) = x^4 + 13x^2 - 48$
find all remaining zeros.

$$x = 4i \quad x = -4i$$

$$(x - 4i)(x + 4i)$$

$$x^2 + 16$$

$$x^2 + 0x + 16 \overline{) x^4 + 0x^3 + 13x^2 + 0x - 48}$$

$$\underline{-x^4 + 0x^3 + 16x^2}$$

$$-3x^2 + 0x - 48$$

$$\underline{+3x^2 + 0x - 48}$$

$$0$$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \pm \sqrt{3}, \pm 4i$$

Given $1 - 2i$ is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$
find all remaining zeros.

Given $1 - 2i$ is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$
find all remaining zeros.

$$x = 1 - 2i \quad x = 1 + 2i$$

$$(x - 1 + 2i)(x - 1 - 2i)$$

$$x^2 - x - 2x + 1 + 2i + 2x - 2i - 4i^2$$

$$x^2 - 2x + 5$$

$$4x^2 + 8x + 13$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 4 \cdot 13}}{2 \cdot 4}$$

$$x = \frac{-8 \pm \sqrt{-144}}{8} = \frac{-8 \pm 12i}{8} = \left[\frac{-2 \pm 3i}{2}, 1 \pm 2i \right]$$

$$\begin{array}{r} 4x^2 + 8x + 13 \\ x^2 - 2x + 5 \overline{) 4x^4 + 0x^3 + 17x^2 + 14x + 65} \\ \underline{-4x^4 + 8x^3 + 20x^2} \\ 8x^2 - 3x^2 + 14x \\ \underline{-8x^2 + 16x^2 + 40x} \\ 13x^2 - 26x + 65 \\ \underline{-13x^2 + 26x + 65} \\ 0 \end{array}$$

Determine the end behavior of the function

$$f(x) = -12x^5 + x^3 + 10x^2 + 8$$

Determine the end behavior of the function

$$f(x) = 13x^4 + 7x^3 - 10x^2 + x - 3$$

Determine the end behavior of the function

$$f(x) = -12x^5 + x^3 + 10x^2 + 8$$

negative odd



$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = -\infty$$

Determine the end behavior of the function

$$f(x) = 13x^4 + 7x^3 - 10x^2 + x - 3$$

positive even



$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

