

Chapter 9 Day 4:

Today you will learn how to rewrite a quadratic in Vertex Form. This process is known as completing the square.

$$y = ax^2 + bx + c \quad \text{Standard Form}$$

$$y = a(x-h)^2 + k \quad \text{Vertex Form}$$

$$y = 2(x-3)^2 + 1$$

$$2(x-3)(x-3) + 1$$

FOIL

$$2(x^2 - 3x - 3x + 9) + 1$$

$$2(x^2 - 6x + 9) + 1$$

$$2x^2 - 12x + 18 + 1$$

$$y = 2x^2 - 12x + 19$$

First I will take you through a problem, then I will give you the steps. This way the steps will make more sense:)

$$y = a(x-h)^2 + k$$

Example: Write this equation in Vertex Form using Completing the Square.

$$y = x^2 - 16x + 15$$

$$y = x^2 - 16x + \frac{64}{2} + 15 - \frac{64}{2}$$

$$y = (x - 8)^2 - 49$$

$$V = (8, -49)$$

Steps to follow:

1. Factor out a GCF
2. Rewrite the problem placing a + ____ and - ____ in the appropriate places.
3. Below this line begin to write a ()²
4. Now fill the parenthesis with the letter used in the problem and 1/2 of the b term.
5. Now square the number that you just found and write it in the blanks above.
6. Combine the last two numbers and write them after your ()²

Ex: $y = x^2 + 10x - 3$

$$y = x^2 + 10x + \frac{25}{2} - 3 - \frac{25}{2}$$

$$y = (x + 5)^2 - 28$$

$$V = (-5, -28)$$

$$y = a(x - h)^2 + k$$

$$V = (h, k)$$

Example: Write this equation in Vertex Form using Completing the Square.

$$y = n^2 - 6n - 72$$

$$y = n^2 - 6n + \frac{9}{2} - 72 - \frac{9}{2}$$

(Handwritten notes: a green arrow points from $\frac{9}{2}$ to $\frac{1}{2}$ with a '2' below it, and a blue arrow points from $\frac{9}{2}$ to the -81 in the next line.)

$$y = (n - 3)^2 - 81$$

$$V = (3, -81)$$

Example: Write this equation in Vertex Form using Completing the Square.

$$f(x) = x^2 - 7x + 1$$

$$\frac{1}{2} \cdot 7 = \frac{7}{2}$$

$$f(x) = x^2 - 7x + \frac{49}{4} + \underbrace{1 - \frac{49}{4}}_{\frac{4}{4}}$$

$$f(x) = \left(x - \frac{7}{2}\right)^2 - \frac{45}{4} \quad (x - 3.5)^2 - 11.25$$

$$V = \left(\frac{7}{2}, -\frac{45}{4}\right)$$

$$(3.5, -11.25)$$

$$\begin{array}{r} 9 \\ 85 \\ 85 \\ \hline 7225 \end{array}$$

$$\begin{array}{r} 3 \\ 25 \\ 25 \\ \hline 625 \end{array} \quad \begin{array}{r} 4 \\ 8.5 \\ 3.5 \\ \hline 12.25 \end{array}$$

Example: Write this equation in Vertex Form using Completing the Square.

$$f(z) = z^2 + 11z + \frac{21}{4}$$

$$f(z) = z^2 + 11z + \frac{121}{4} + \frac{21}{4} - \frac{121}{4}$$

$$f(z) = \left(z + \frac{11}{2}\right)^2 - \frac{100}{4}$$

$$f(z) = \left(z + \frac{11}{2}\right)^2 - 25$$

$$V = \left(\frac{-11}{2}, -25\right)$$

Example: Change $f(x)$ from Vertex Form into Standard Form.

$$f(x) = 3(x-5)^2 - 72$$

$$3(x-5)(x-5) - 72$$

$$3(x^2 - 5x - 5x + 25) - 72$$

$$3(x^2 - 10x + 25) - 72$$

$$3x^2 - 30x + 75 - 72$$

$$f(x) = 3x^2 - 30x + 3$$

$$x^2 + 14x + \underline{49}$$

$$7^2$$