Chapter 7 Review Part 1 No Calculator Name________________

1.Find the general solution to the exact differential equation.

$$\frac{dy}{dx} = \sin x - e^{-x} + 8x^3$$

$y = -\cos x + e^{-x} + 2x^4 + C$

2. Solve the initial value problem explicitly.

5. $\frac{dy}{dx} = 1 + x + \frac{x^2}{2}, \quad y(0) = 1$

$y = x + \frac{1}{2}x^2 + \frac{x^3}{6} + C$ $y = x + \frac{x^2}{2} + \frac{x^3}{3} + 1$

$1 = 0 + C \quad C = 1$

3. Find an integral equation $y = \int_a^x f(t)dt + b$ such that $dy/dx = \sin^3 x$ and $y = 5$ when $x = 4$.

$y = \int_4^x \sin^3 x\, dx + 5$

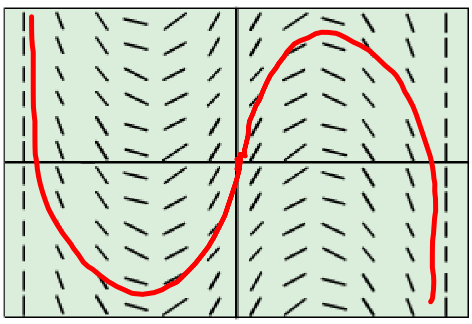
4. Evaluate the integral

x^{-2}

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}\ (x > 0)$$

$y = \ln x + x^{-1} + C$ $\quad$ $y = \ln x + \frac{1}{x} + C$

5. ***Sketching Solutions*** Draw a possible graph for the function $y = f(x)$ with slope field given in the figure that satisfies the initial condition $y(0) = 0$. $(0,0)$



[–10, 10] by [–10, 10]

6. Evaluate the integral

$$\frac{1}{3}\int \frac{3\,dx}{\sqrt[3]{3x+4}}$$

$u = 3x+4 \quad du = 3dx$

$\frac{1}{3}\int u^{-\frac{1}{3}}\,du \quad \frac{1}{3}\cdot\frac{3}{2}u^{\frac{2}{3}} \quad \frac{1}{2}u^{\frac{2}{3}}+C$

$\frac{1}{2}(3x+4)^{\frac{2}{3}}+C$

7. Evaluate the integral

$$\frac{1}{2}\int \frac{2\,x\,dx}{x^2+1}$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|+C \quad \boxed{\frac{1}{2}\ln(x^2+1)+C}$

8. Evaluate the integral

$$-\int -\sqrt{\cot x}\,\csc^2 x\,dx$$

$u = \cot x \quad du = -\csc^2 x\,dx$

$-\int u^{\frac{1}{2}}\,du \quad -\frac{2}{3}u^{\frac{3}{2}}+C$

$$\boxed{\begin{gathered}-\frac{2}{3}(\cot x)^{\frac{3}{2}}+C \\ -\frac{2}{3}\cot^{\frac{3}{2}}x+C\end{gathered}}$$

9. Evaluate the integral

$$-\frac{1}{2}\int \frac{-2\sin(2t+1)}{\cos^2(2t+1)}\,dt$$

$u=\cos(2t+1)$

$du=-2\sin(2t+1)dt$

$-\frac{1}{2}\int u^{-2}du$

$-\frac{1}{2}(-1)u^{-1} = \frac{1}{2}u^{-1}+C$

$$\boxed{\frac{1}{2}(\cos(2t+1))^{-1}+C \qquad \frac{1}{2\cos(2t+1)}+C}$$

10. In Exercises 47–52, use the given trigonometric identity to set up a u-substitution and then evaluate the indefinite integral.

51. $\int \tan^4 x\,dx, \quad \tan^2 x = \sec^2 x - 1$

$\tan^4 x = \tan^2 x \cdot \tan^2 x$

$\int \tan^2 x(\sec^2 x - 1)dx$

$\int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$

$u=\tan x \quad du=\sec^2 x\,dx$

$\int u^2 du$ $\qquad -\int(\sec^2 x - 1)dx$

$\frac{1}{3}u^3$

$\frac{1}{3}\tan^3 x$ $\qquad -[\tan x - x]+C$

$$\boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$$

11. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b).

$$\int_0^{\pi/2} 5\sin^{3/2} x \cos x \, dx$$

$u = \sin x \quad du = \cos x \, dx$

$5\int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}} x \cos x \, dx$

$u(0) = 0$

$u(\frac{\pi}{2}) = 1$

$5\int_0^1 u^{\frac{3}{2}} \, du$

$5 \cdot \frac{2}{5} u^{\frac{5}{2}} \Big|_0^1$

$2\left[1^{\frac{5}{2}} - 0^{\frac{5}{2}}\right]$

$2[1-0] = \boxed{2}$

12. Evaluate the definite integral by making a u-substitution and integrating from u(a) to u(b).

$-\frac{1}{2}\int_0^1 r\sqrt{1-r^2}\,dr$ -2

$u = 1-r^2 \quad du = -2r\,dr$

$u(0) = 1 \quad u(1) = 0$

$-\frac{1}{2}\int_1^0 u^{\frac{1}{2}}\,du$

$-\frac{1}{2}\frac{2}{3}u^{\frac{3}{2}}\Big|_1^0$

$-\frac{1}{3}\left[0^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] \quad -\frac{1}{3}(-1) = \boxed{\frac{1}{3}}$

13. Evaluate the integral

$\int 3t\,e^{2t}\,dt$

$u = 3t \qquad dv = e^{2t}\,dt$

$du = 3dt \qquad v = \frac{1}{2}e^{2t}$

$\rightarrow \int u\,dv = uv - \int v\,du$

$= 3t\left(\frac{1}{2}e^{2t}\right) - \int \frac{1}{2}e^{2t}(3)\,dt$

$= \frac{3t}{2}e^{2t} - \frac{3}{2}\frac{1}{2}e^{2t} + C$

$\boxed{= \frac{3t}{2}e^{2t} - \frac{3}{4}e^{2t} + C}$

14. Use tabular integration to find the antiderivative.

$$\int x^3 \cos x \, dx$$

x^3	$+$	$\cos x$
$3x^2$	$-$	$\sin x$
$6x$	$+$	$-\cos x$
6	$-$	$-\sin x$
0		$\cos x$

$$x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$$